X-ray Optics

X-Ray Astronomy School VI

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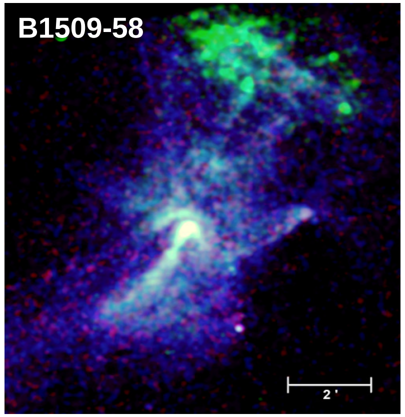
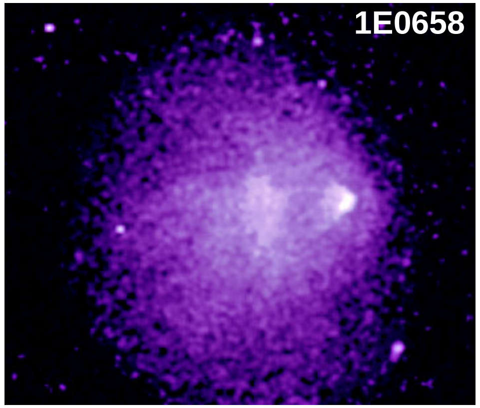
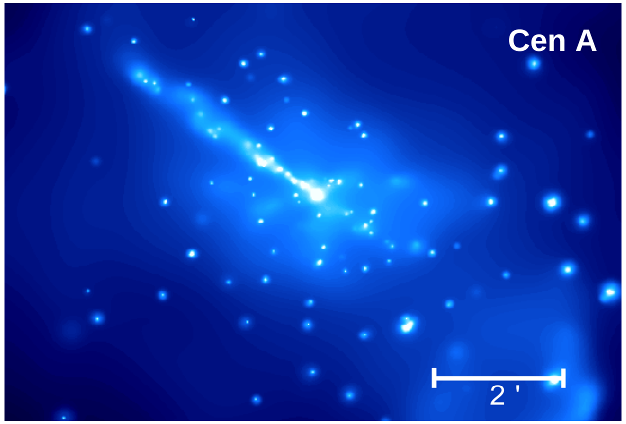
SAO/CXC

How do we form an X-Ray Image?



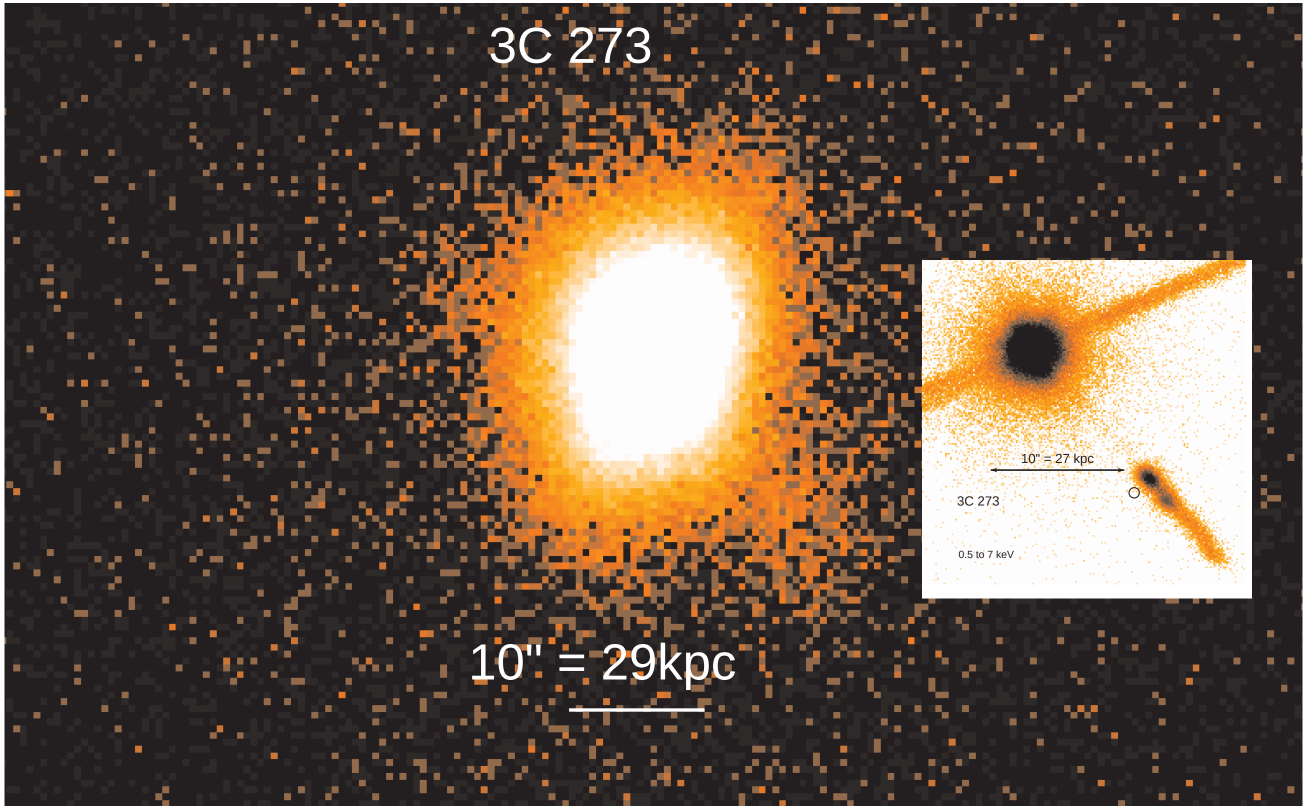




Why do we Use X-Ray Optics?

1. To achieve the best, 2-dimensional angular resolution

- Distinguish nearby sources, different regions of the same source
- Use morphology to apply intuition and choose models.

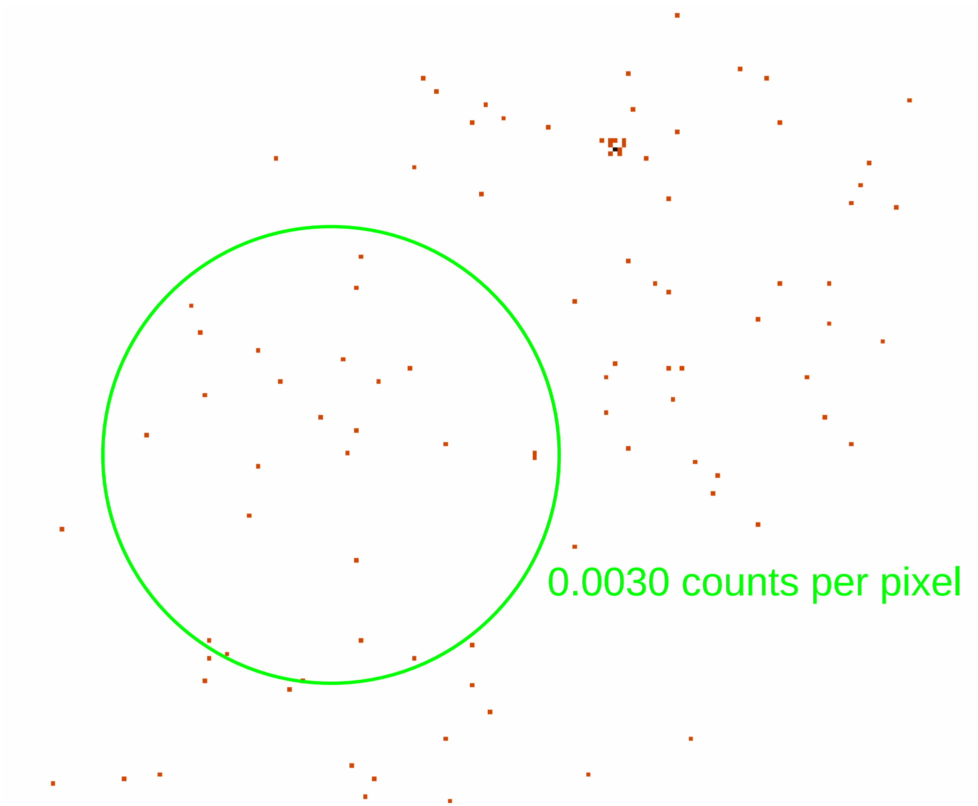


Why do we Use X-Ray Optics?

2. As a collector to “gather” weak fluxes of photons

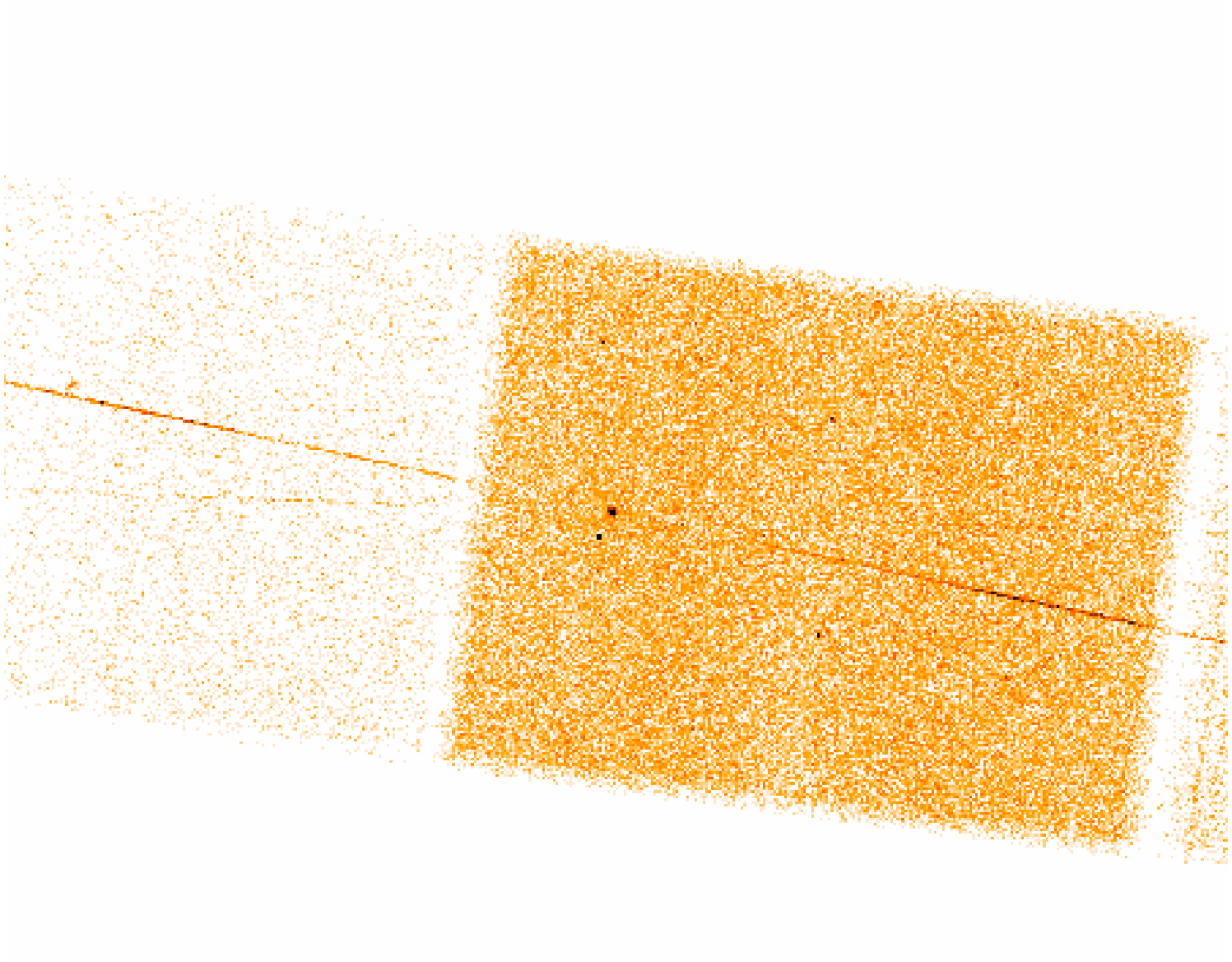
3. As a concentrator, so that the image photons interact in such a small region of the detector that background is negligible or small

4. To simultaneously measure both the sources of interest, and the contaminating background using other regions of the detector



Why do we Use X-Ray Optics?

5. To serve with high spectral resolution dispersive spectrometers such as transmission or reflection gratings.



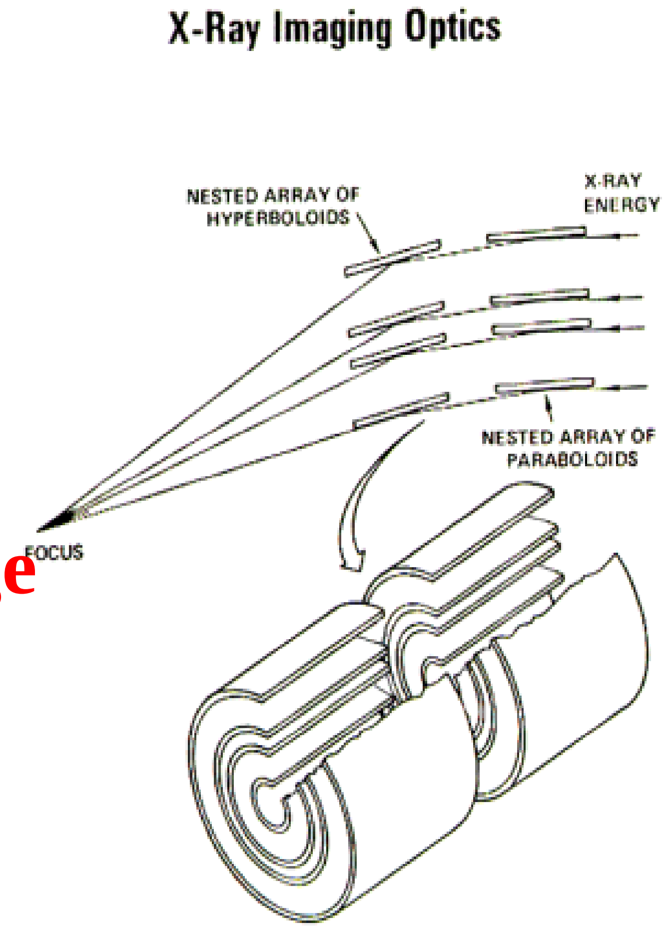
X-Ray Optics

1. We must make the X-rays Reflect

- Total External Reflection
- Fresnel’s Equations

2. We must make the X-rays form an Image

- Mirror Figure
- Scattering

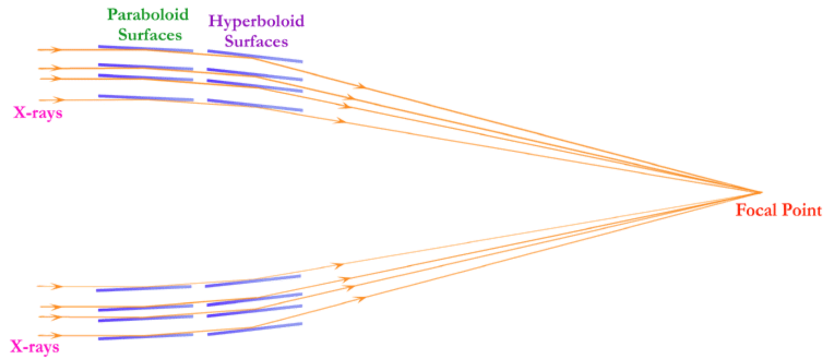


X-Ray Reflection: Zero Order Principles:

Refs:

Gursky, H., and Schwartz, D. 1974, in "X-Ray Astronomy," R. Giacconi and H. Gursky eds., (Boston: D. Reidel) Chapter 2, pp 71-81;

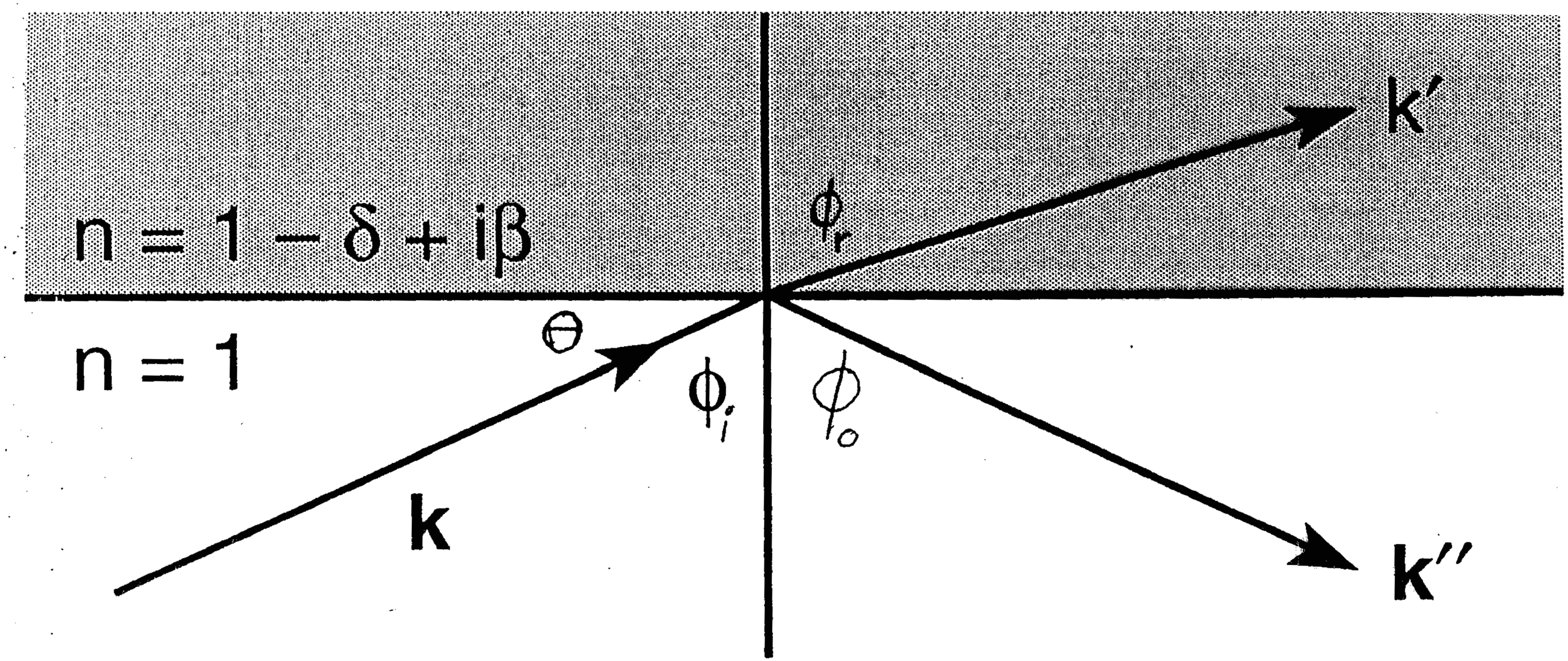
Aschenbach, B. 1985, Rep. Prog. Phys. 48, 579.



X-rays reflect at small grazing angles.

An analogy is skipping stones on water.

Scattering of any wave by an ensemble of electrons is coherent only in very special directions; namely, the familiar Angle of Incidence equals Angle of Reflection, $\phi_i = \phi_o$.



We have Snell's law for refraction, $\sin \phi_r = \sin \phi_i / n$ or $\cos \theta_r = \cos \theta_i / n$

ϕ is the standard angle of incidence from the surface normal,

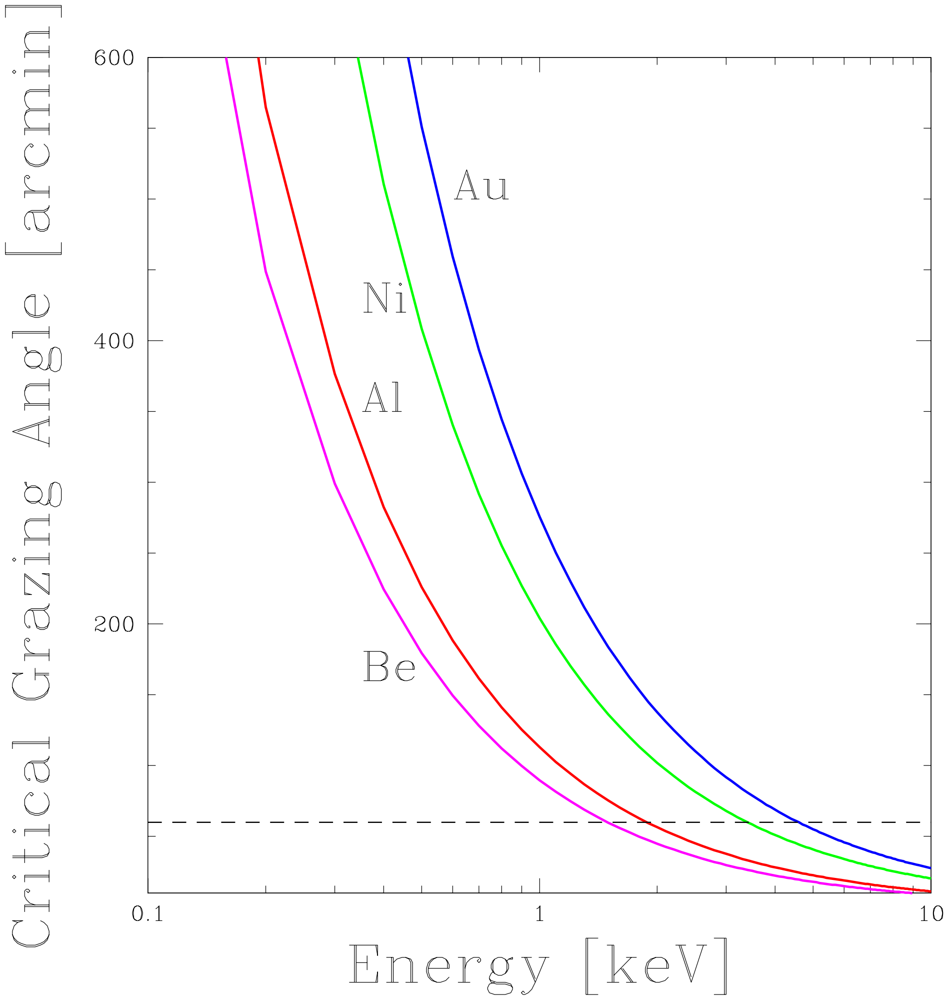
θ is the grazing angle from the surface.

The complex index of refraction is $n = 1 - \delta + i\beta$.

We have used the subscripts *i* for the incident photon, *o* for the reflected or outgoing photon, and *r* for the refracted photon.

Figure from Atwood, D. 1999, "Soft X-rays and Extreme Ultraviolet Radiation: Principles and Applications, (http://www.coe.berkeley.edu/AST/sxreuv)

Critical Grazing Angle



Total external reflection works for $\theta_i \leq \theta_c$

since cos θ_c/n =cos θ_r =1 .

The limiting condition is cos θ_c=n.

cos $\theta_c \approx 1 - \theta_c^2/2 = 1 - \delta, \therefore \theta_c = \sqrt{(2\delta)}$.

Away from absorption edges, $\delta = r_0 \lambda^2 N_e/(2\pi)$. $\therefore \theta_c \propto \sqrt{(Z)}/E$

- **The critical angle decreases inversely proportional to the energy.**
- **Higher Z materials reflect higher energies, for fixed grazing angles.**
- **Higher Z materials have a larger critical angle at any energy.**

SPECIAL TOPIC: Index of Refraction

Attwood, D. 1999, "Soft X-rays and Extreme Ultraviolet Radiation: Principles and Applications, (http://www.coe.berkeley.edu/AST/sxreuv)

Consider a plane wave, $E(r,t) = E_0\, e^{-i(\omega t - k\cdot r)}$.

The velocity, ω/k, is c in vacuum, and is c/n = ω/k in a material.

Substituting k = (ω/c) n in the reflecting medium, n=1-δ+iβ

$E(r,t) = E_0\, e^{-i[\omega t - (\omega/c)(1-\delta+i\beta)r]} = E_0\, e^{-i\omega(t-r/c)}\; e^{-i(\omega\delta/c)r}\; e^{-(\omega\beta/c)r}$.

The roles are now clear: we have the wave which would propagate in vacuum, a phase shift caused by the small real deviation from unity, and an attenuation caused by the imaginary part of the index of refraction.

X-Ray Reflection: Fresnel Equations

The Fresnel equations give the correct amplitudes of reflection (and refraction) for a plane wave incident on an infinitely smooth surface.

Use Maxwell's equations, keeping the components of $E_{||}$ and $H_{||}$ and $D_{\perp}$ and $B_{\perp}$ continuous across the interface.

We need to consider polarization to apply the boundary conditions. The reflected wave will have the amplitude

r_p for the parallel component

r_s for the perpendicular component of the electric vector.

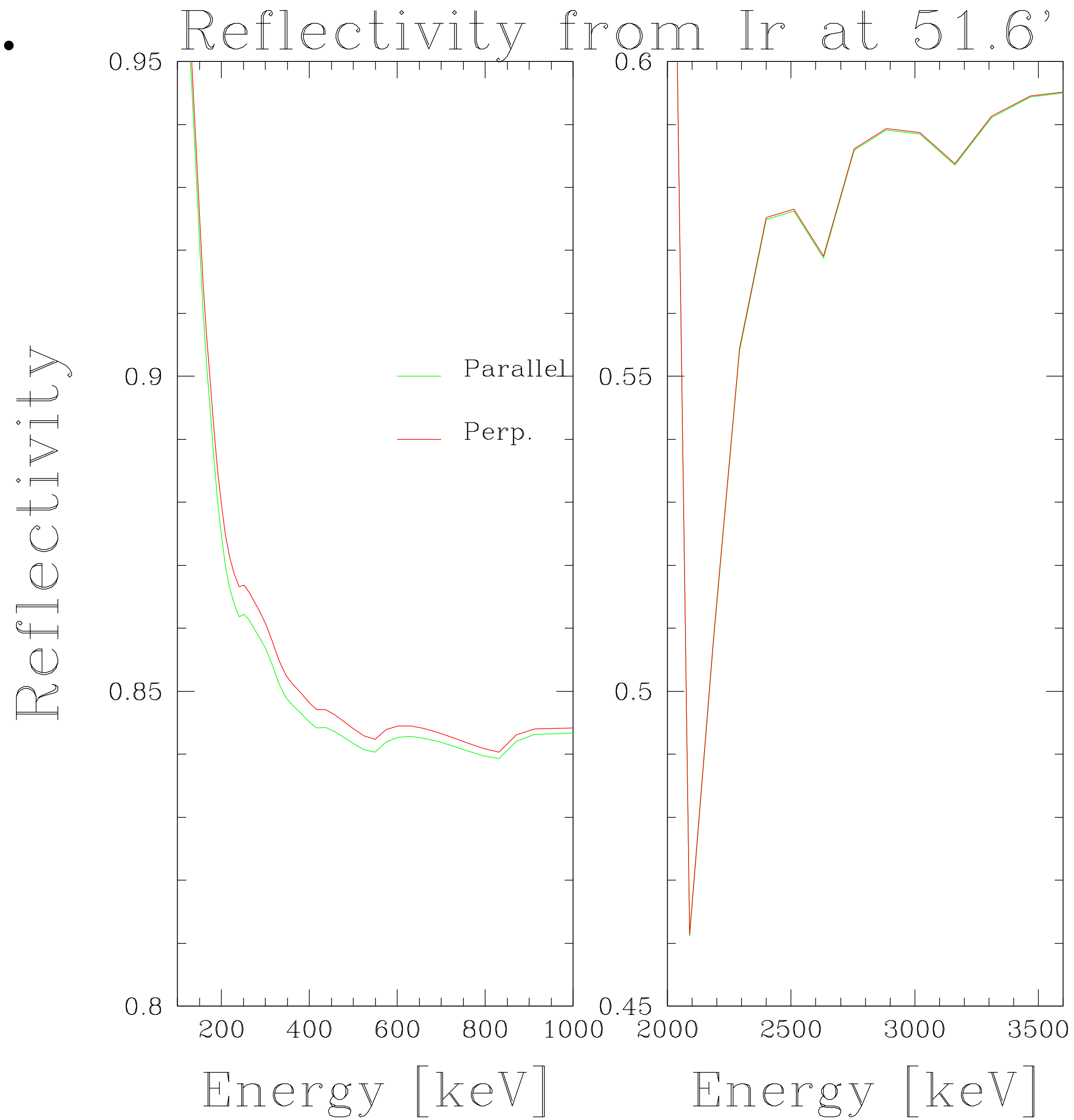
The squared amplitudes of the complex numbers r_p and r_s are the actual reflectivities.

For unpolarized X-rays the reflectivity is just $(|r_p|^2+|r_s|^2)/2$

$$r_p = (n^2 \sin\theta - (n^2 - \cos^2\theta)^{1/2}) / (n^2 \sin\theta + (n^2 - \cos^2\theta)^{1/2})$$

$$r_s = (\sin\theta - (n^2 - \cos^2\theta)^{1/2}) / (\sin\theta + (n^2 - \cos^2\theta)^{1/2})$$

and $r_p \simeq r_s$ for X-rays, since $n \simeq 1$.



X-Ray Reflection: NOT the end of the Story

Three Significant effects remain:

1. The surfaces are not infinitely smooth. This gives rise to the complex subject of X-ray scattering. Scattering cannot be treated *exactly*, one must consider a statistical description of the surface roughness.

Key Features:

- **Scattering increases as E^2**
- **In plane scattering dominates by factor $1/\sin\theta$**

Scattering

Features:

1. Scattering is predominantly in the plane of the incident X-ray and the normal to the surface. Out of plane scattering is less by a factor sin(α).

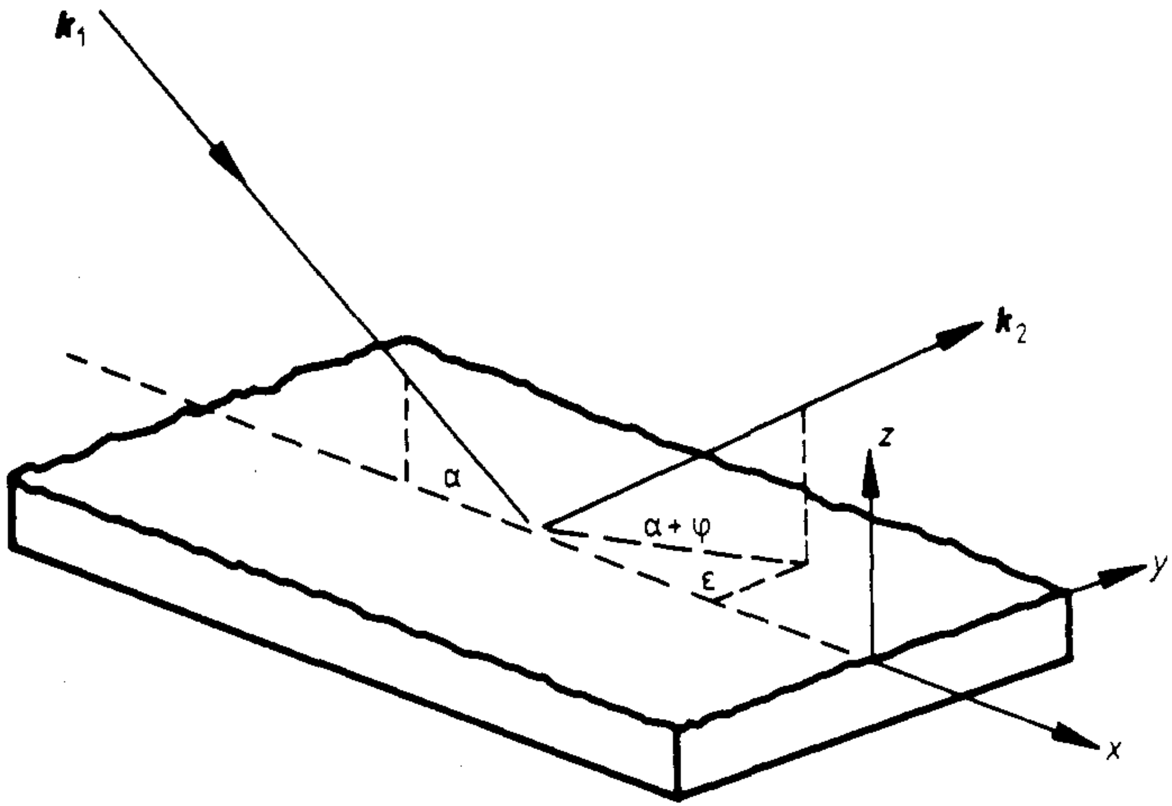
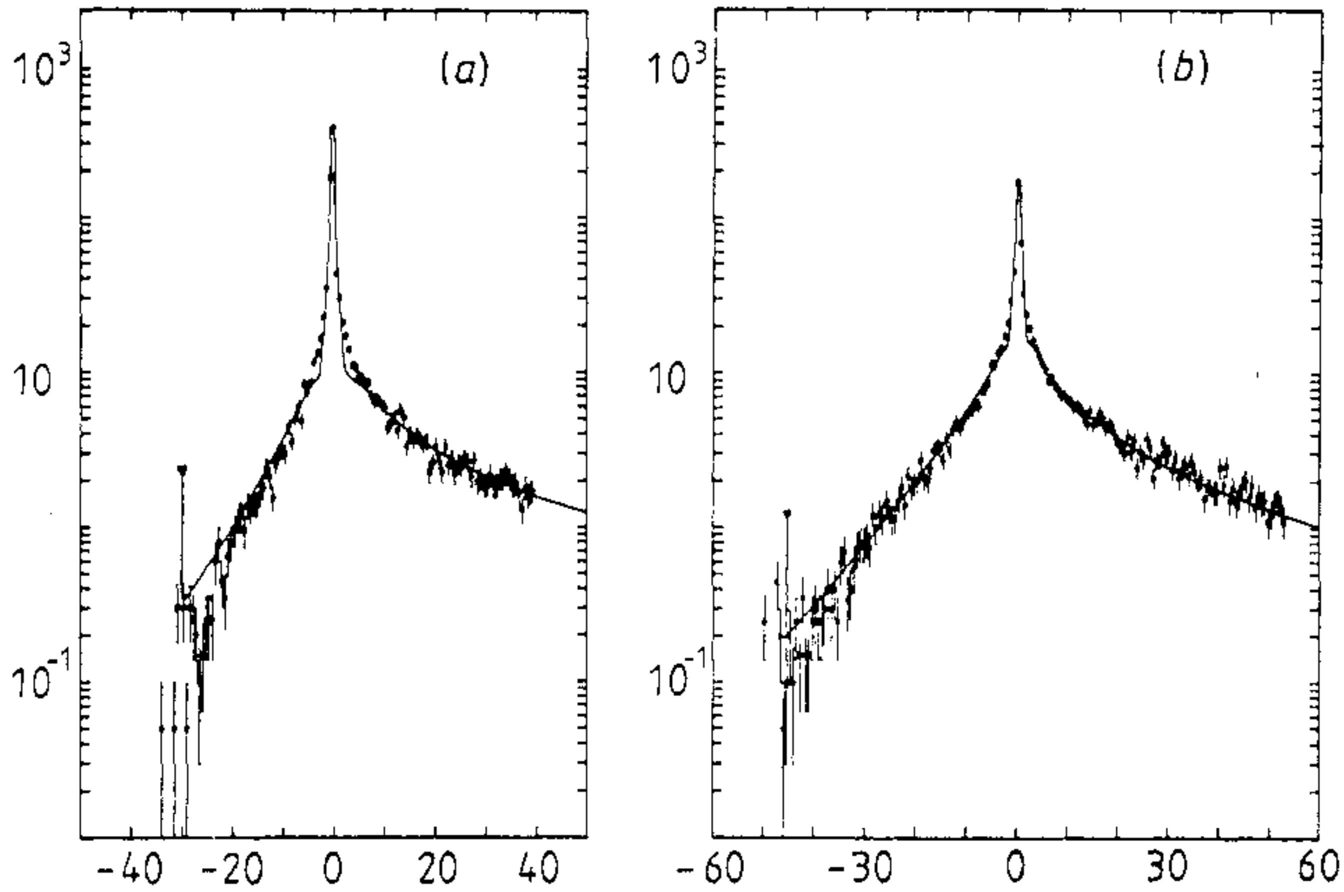


Figure 1. Scattering geometry. $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ denote the wavevector of the incident and scattered ray, respectively.

Scattering

Features:

2. Scattering is asymmetric. Backward scattering can be no more than -α, which would take the ray into the surface. Forward scattering is unlimited.



Scattering Angle [arcmin]

Scattering

Aschenbach, B. 1985, Rep. Prog. Phys. 48, 579; Zhao, P. & VanSpeybroeck, L. P. 2002, SPIE 4844.

Scattering theory treats irregularities in the surface height h as random, characterized by a power spectral density function

$$2W_1(f)=\left| \int e^{i2\pi x f}\, h(x)\, dx \right|^2$$

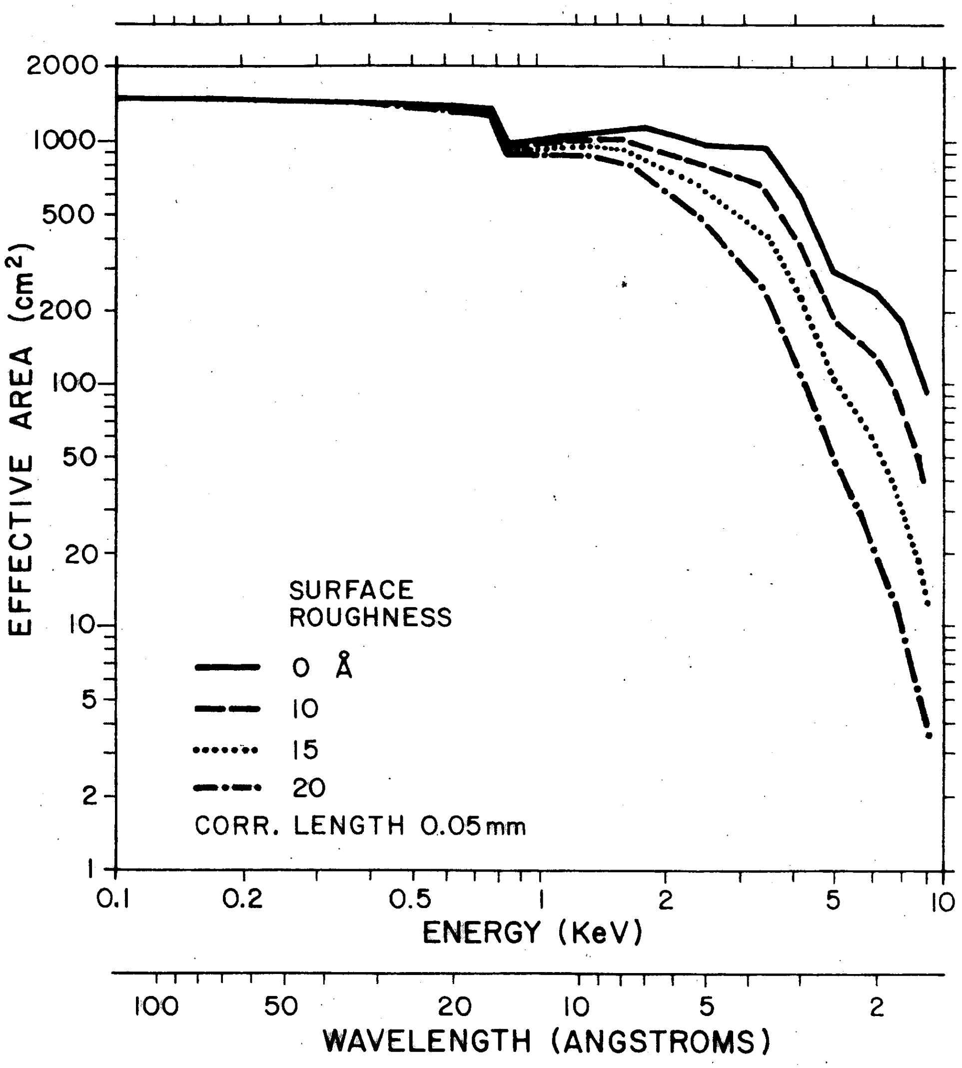
For sufficiently smooth surfaces scattering can be considered as diffraction, so that light of wavelength λ is scattered due to irregularities with spatial frequency f according to the usual grating equation:

$f = \epsilon \sin \alpha/\lambda$, where α is the grazing angle and ϵ the scattering angle.

The scattered intensity, relative to the total power in the focal plane, is

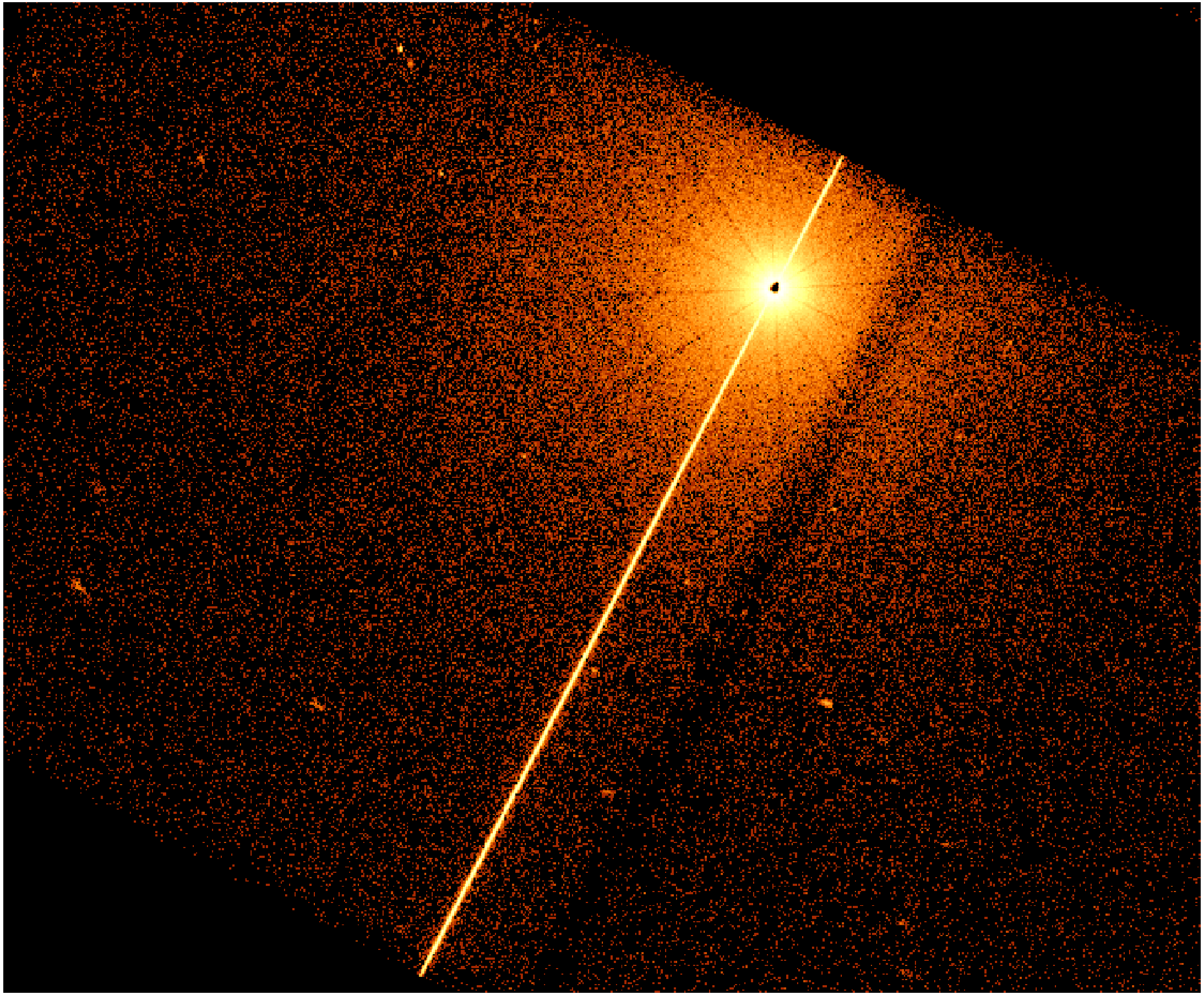
$$\psi(\epsilon)=2W_1(f)\, 8\pi(\sin \alpha)^4/f\, \lambda^4$$

Scattering is Proportional to E^2



For a Gaussian distribution of surface heights, and no correlation of roughness with direction, the relative total scattered intensity is $1\text{-}e^{-(4\pi\sigma \sin\alpha/\lambda)^2} \sim (4\pi\sigma \sin\alpha/\lambda)^2$ when the exponent is small. The rms roughness of the surface, σ is defined as $\sigma^2 = \int 2W_1(f)\,df$.

In plane scattering dominates by a factor 1/sin θ



X-Ray Reflection: NOT the end of the Story

2. We generally do not have a perfect interface from a vacuum to an infinitely thick reflecting layer. We must consider:

- **The mirror substrate material; e.g., Zerodur for Chandra**
- **A thin binding layer, e.g., Chromium, to hold the heavy metallic coating to the glass**
- **The high Z metal coating; e.g., Ir for Chandra**
- **An unwanted but inadvertent overcoat of molecular contaminants**

Feature: Interference can cause oscillations in reflectivity.

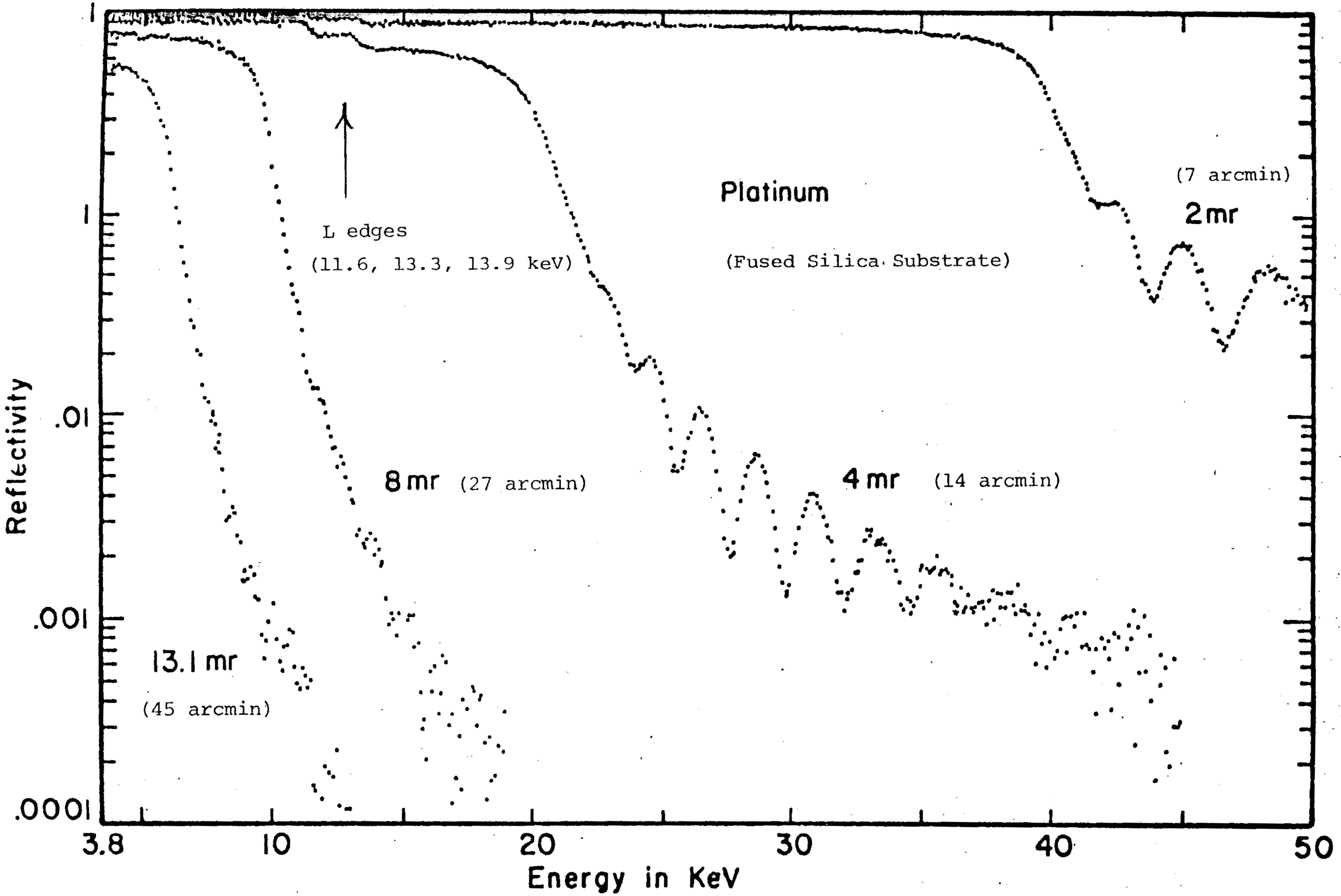
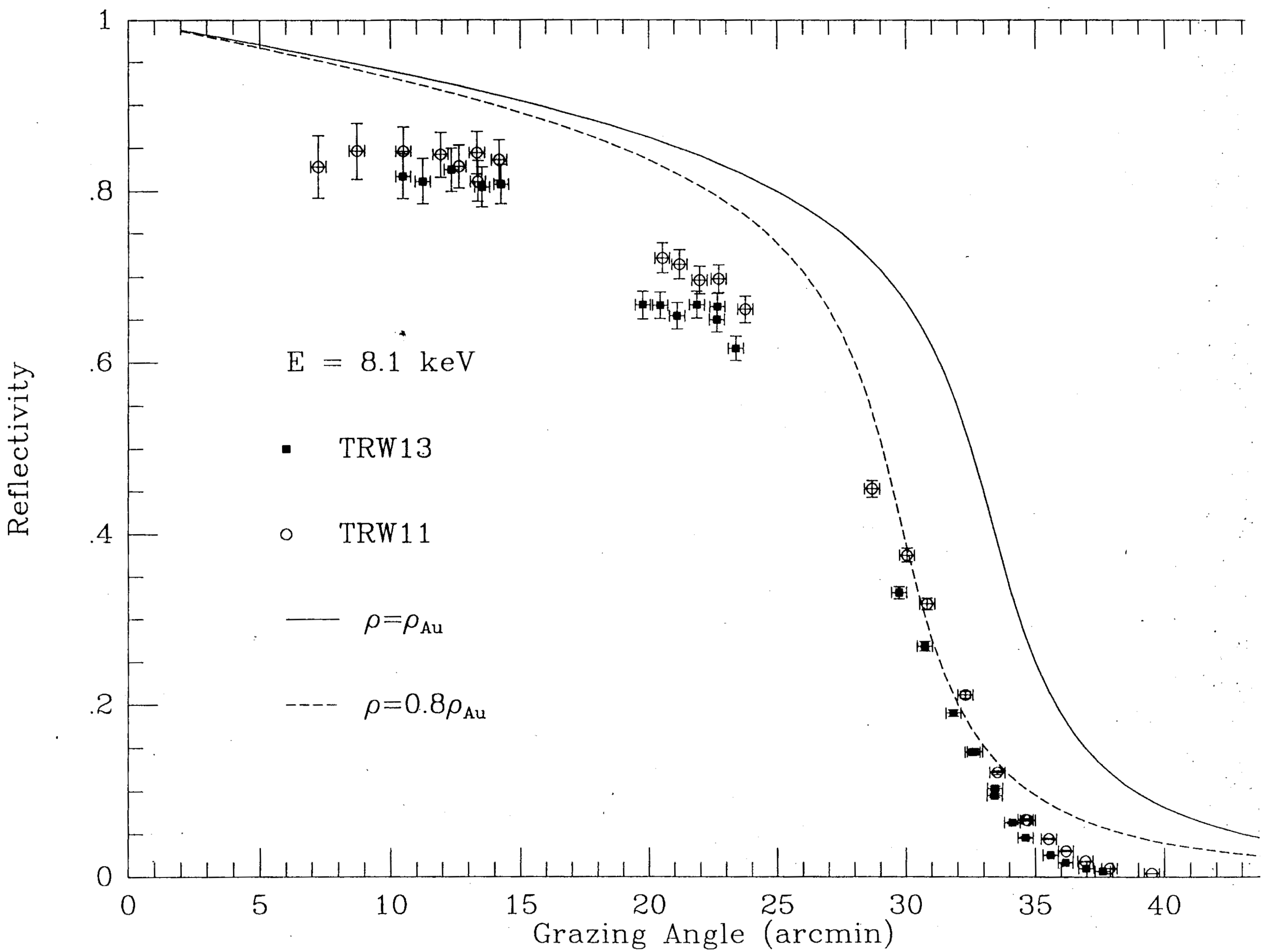


Figure 3.2 The reflectivity of a platinum coated mirror as a function of energy and grazing angle.

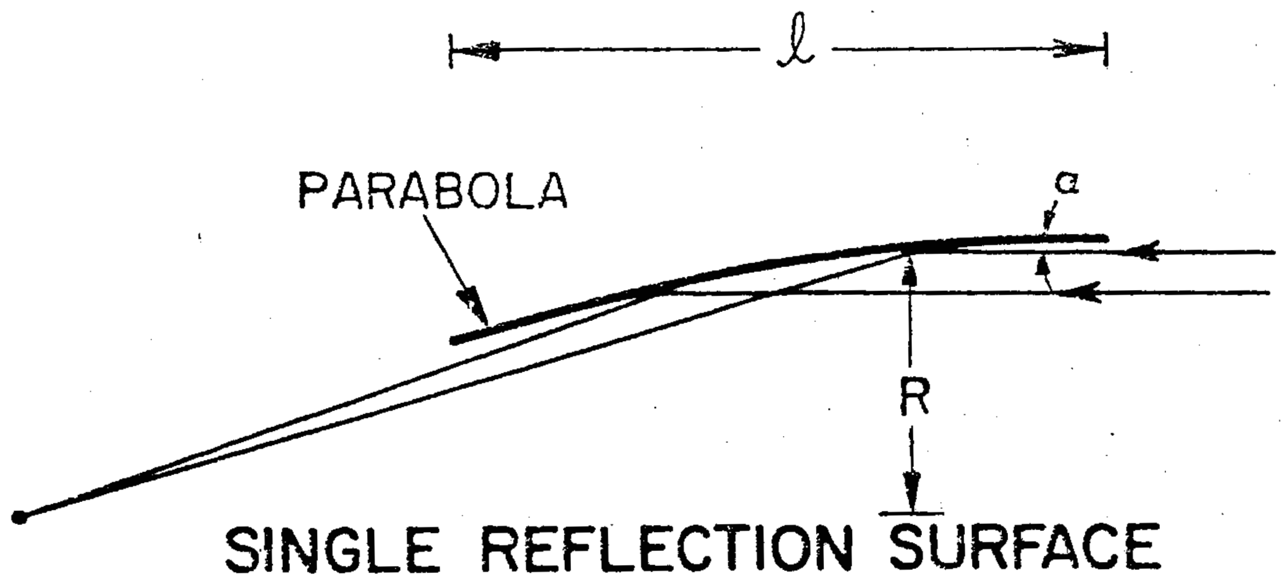
3. Preparation of coating affects reflectivity through the dependence on density.



X-Ray Mirrors: Parabola

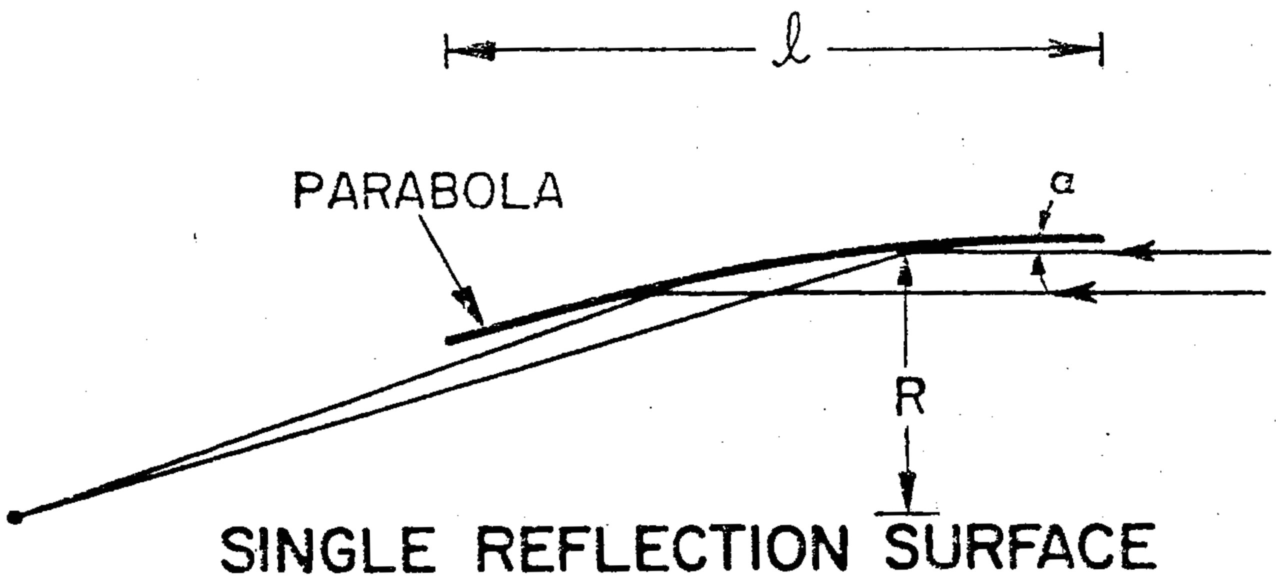
Rays parallel to the parabola axis are focused to a point.

Off-axis, the blur circle increases linearly with the angle.



An incoming ray will hit the parabola at an angle α =arctan($\frac{dR}{dz}$) and be diverted through an angle 2α. One verifies that $R^2=4fz$ satisfies the differential equation tan 2α= R/(z-f), where f is the distance from the origin to the focus, and can be set, e.g., by choosing that the mirror segment, of length ℓ, have a specific grazing angle α at a radius R.

Focal Length: F=R/2α



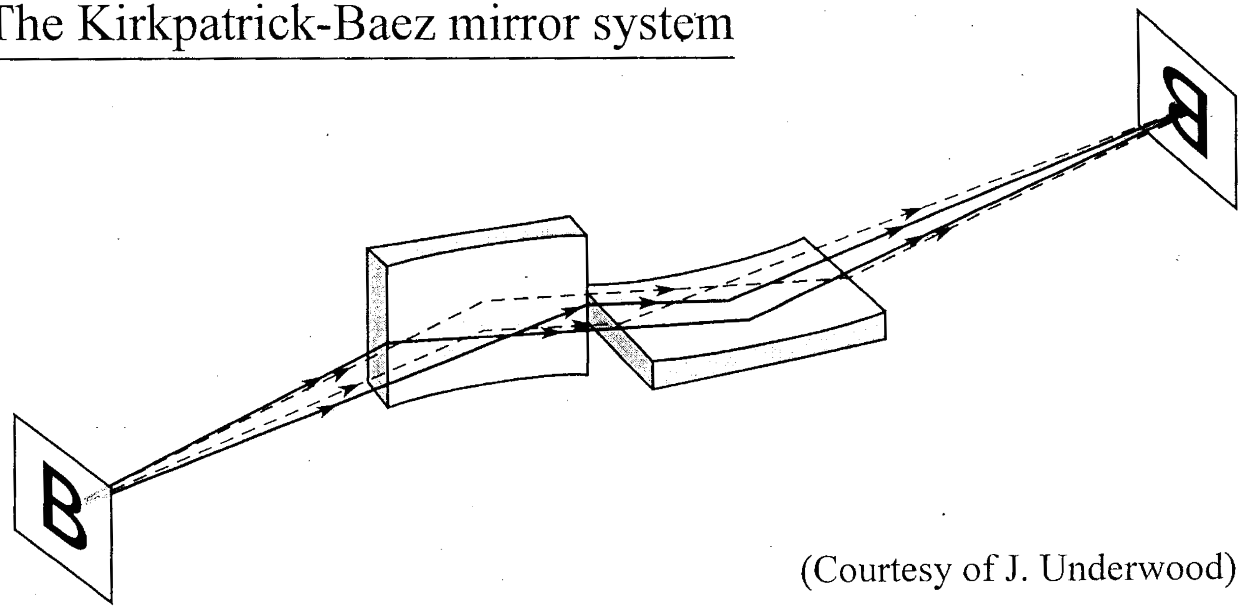
There are two uses for the parabola:

1. Translate the parabola a distance *h* to form a plate, curved in one dimension. This will produce one dimensional focussing of a point to a line, while another plate at right angles can focus in the other dimension.

Parabolic Plate Area = $\alpha\ell h$

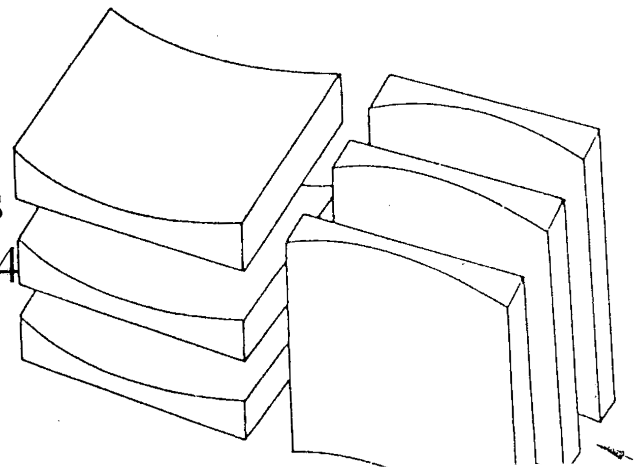
Focusing with Curved, Glancing Incidence Optics

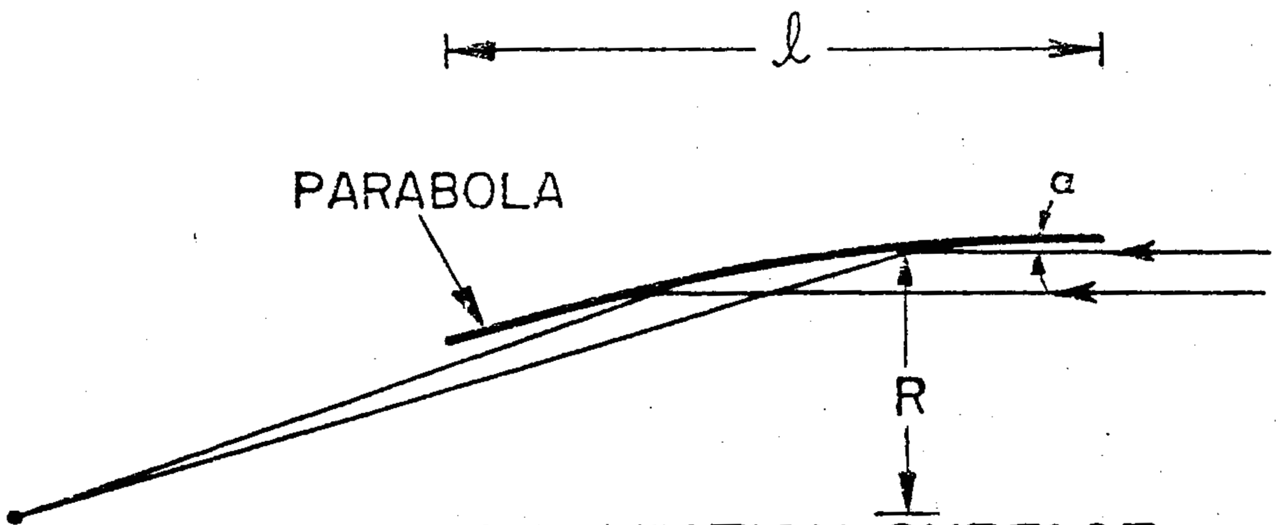
The Kirkpatrick-Baez mirror system



(Courtesy of J. Underwood)

- Two crossed cylinders (or spheres)
- Astigmatism cancels
- Fusion diagnostics
- Common use in synchrotron radiation beamlines
- See hard x-ray microprobe, chapter 4, figure 4.14





SINGLE REFLECTION SURFACE

There are two uses for the parabola:

2. Form a Paraboloid of revolution, and use with an hyperboloid.

Paraboloid Area=$2\pi R\alpha\ell$

X-Ray Mirrors: Wolter's Configurations

Wolter, H. 1952, *Ann. Physik* 10, 94;*ibid.* 286;

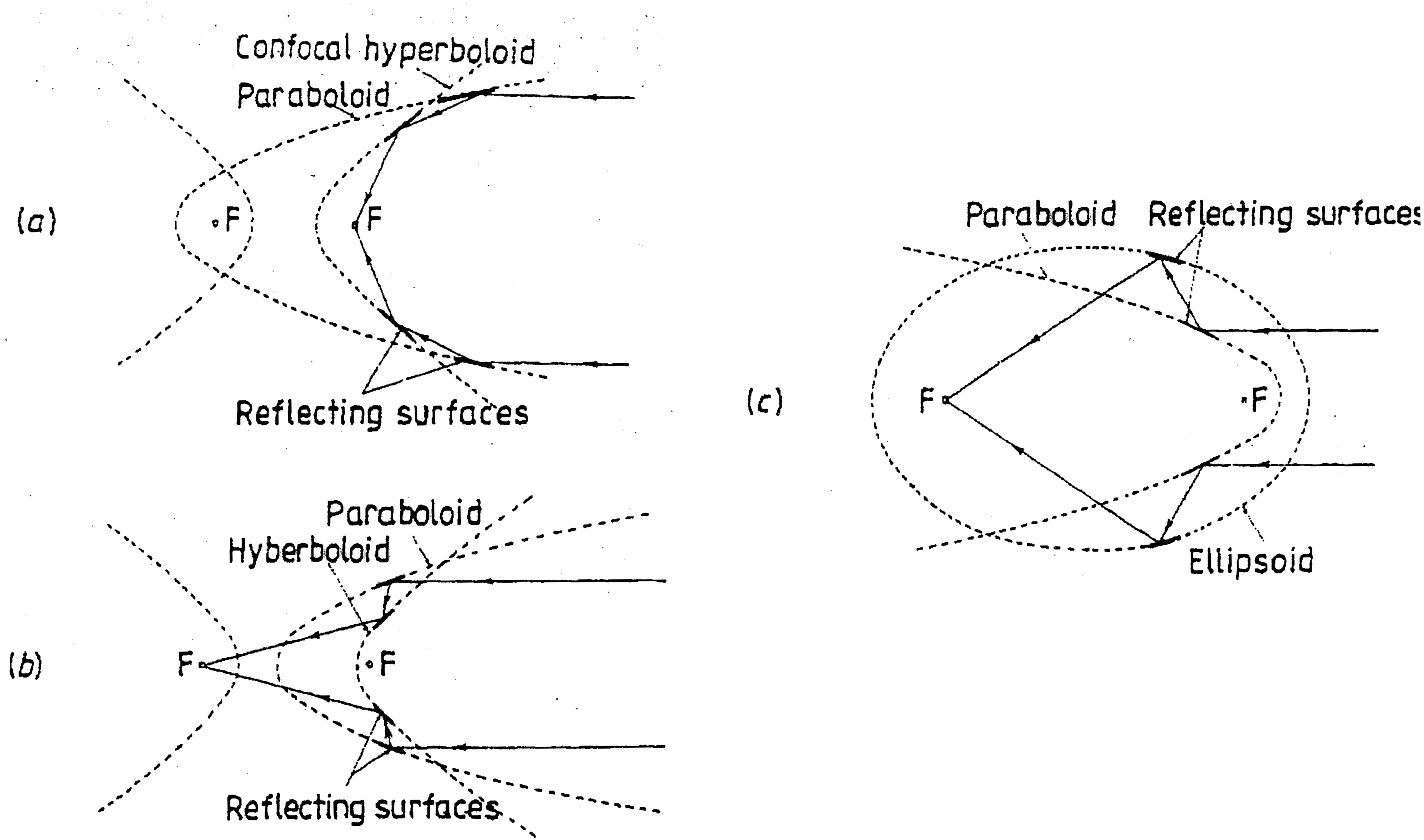
Giacconi, R. & Rossi, B. 1960, *J. Geophys. Res.* 65, 773

A Paraboloid produces a perfect focus for on-axis rays.
Off-axis it gives a coma blur size proportional to the distance off-axis.
Wolter's classic paper proved two reflections were needed, and considered configurations of conics to eliminate coma.
Basic Principle: The optical path to the image must be identical for all rays incident on the telescope, in order to achieve perfect imaging.

Wolter derived three possible Geometries.

The Wolter Geometries

B Aschenbach



X-Ray Mirrors: Wolter's Configurations

The Paraboloid-Hyperboloid is overwhelming most useful in cosmic X-ray astronomy:

- **Shortest Focal length to aperture ratio. This has been a key discriminant as we are always trying to maximize the collecting area to detect weak fluxes, but with relatively severe restrictions on diameter (and length) imposed by available space vehicles.**

- **For resolved sources, the shorter focal length concentrates a given spatial element of surface brightness onto a smaller detector area, hence gives a better signal to noise ratio against the non-X-ray detector background. (*BUT, puts greater demand on having a detector with very small spatial resolution in order to sample the image.***

X-ray Focus

A Grazing Incidence telescope acts as a thin lens. As the telescope tilts about small angles, the image of a point source near the axis remains invariant in space.

- **This is what allows us to convert a linear distance y between two images to an angular distance θ = y/F.**
- **This shows that the optimum focal surface is a bowl shape, sitting on the flat plane perpendicular to the optical axis.**

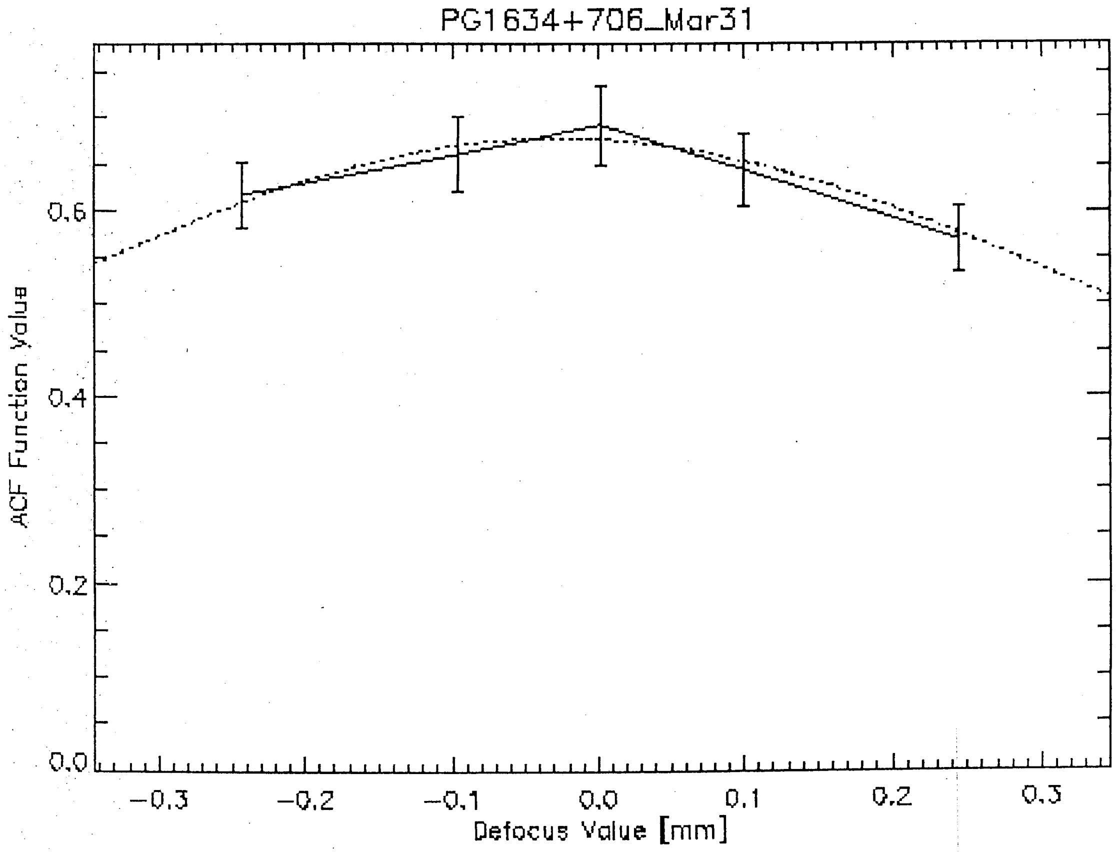
In a Wolter I system the rays from an on-axis point source converge to focus in a cone of half-angle four times the grazing angle.

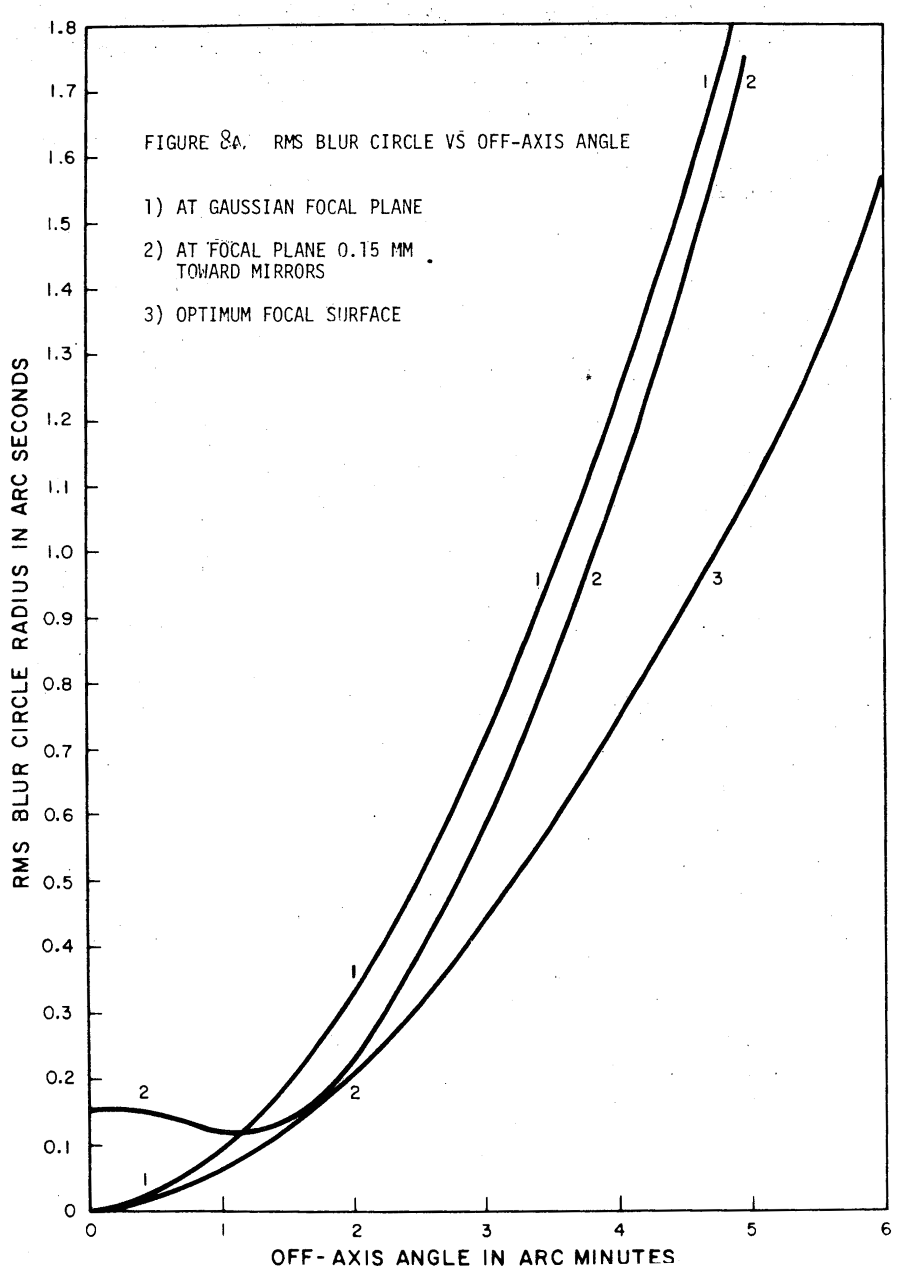
X-ray Focus

***Chandra* cone angles are 3.417°, 2.751°, 2.429°, and 1.805°.**

For a 0.1” contribution to the on-axis blur due to imprecision of the focus, we must be able to focus within

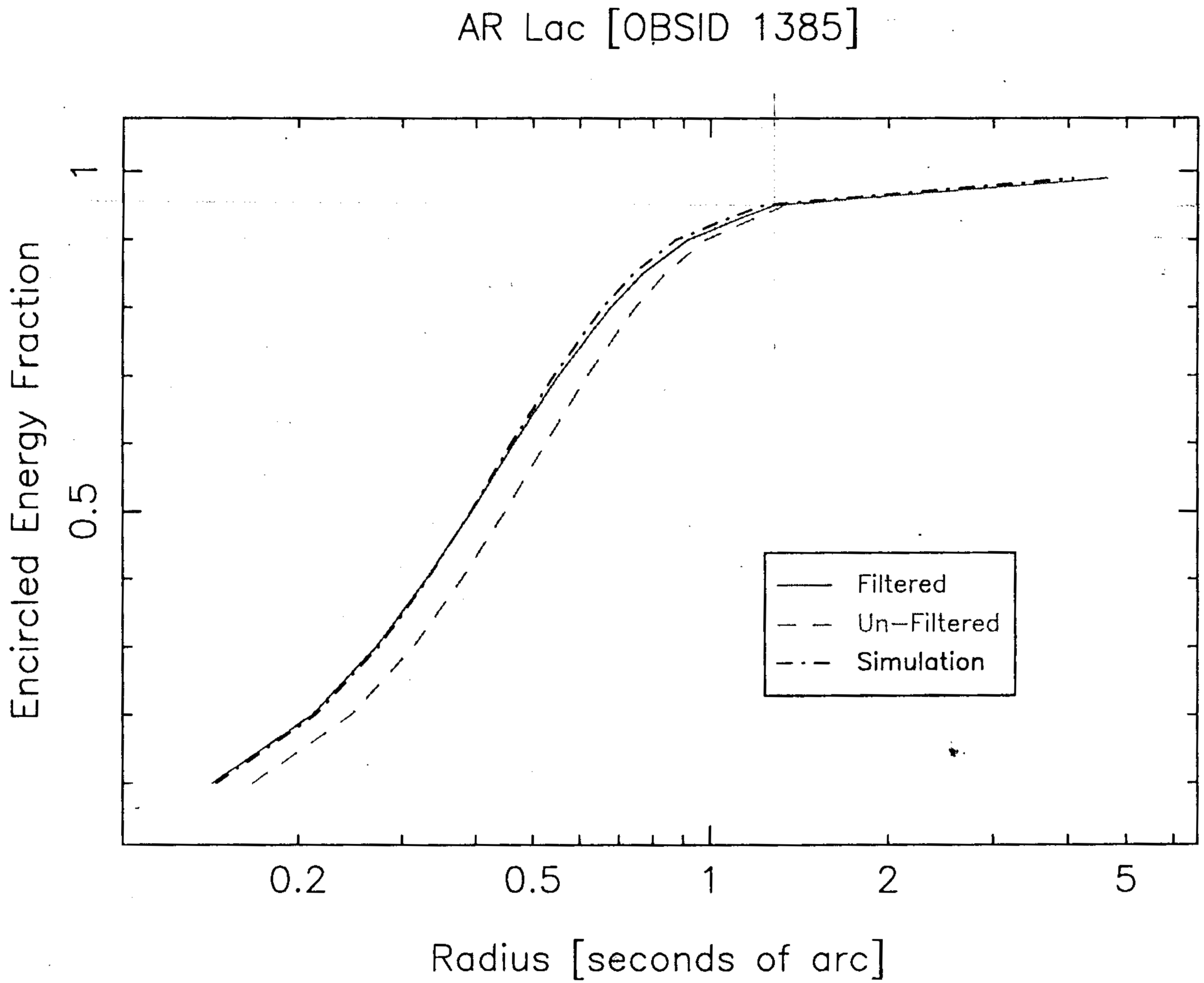
5 μm/tan3.4°= 85μm.

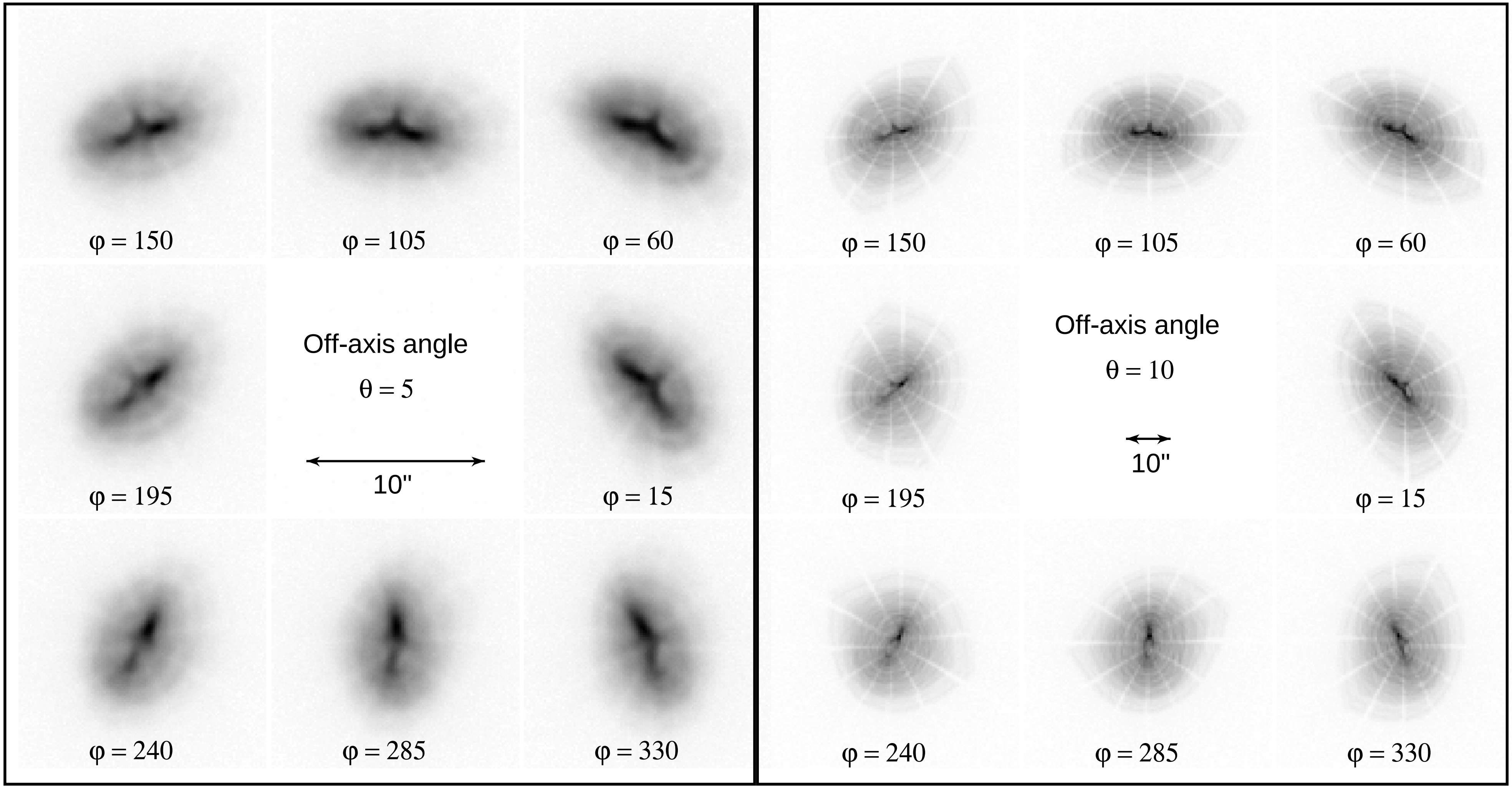




Because the optimal focal plane is curved toward the mirror, and because optical imperfections in mirror figure and mechanical tolerances in aligning the pieces of glass prevent perfect imaging on axis, it is generally advantageous to position a flat imaging detector slightly forward of the ideal on-axis focus.

Encircled energy results, relative to flux in a 10$''$ radius aperture:



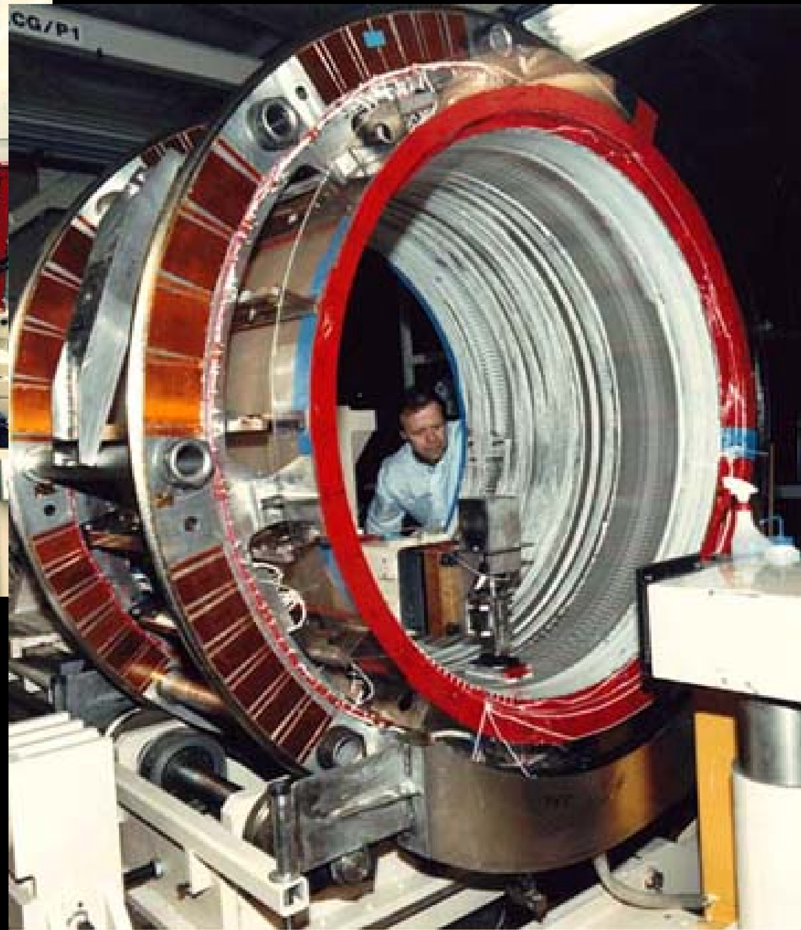


Chandra point spread function, 5 and 10 arc minutes off-axis.

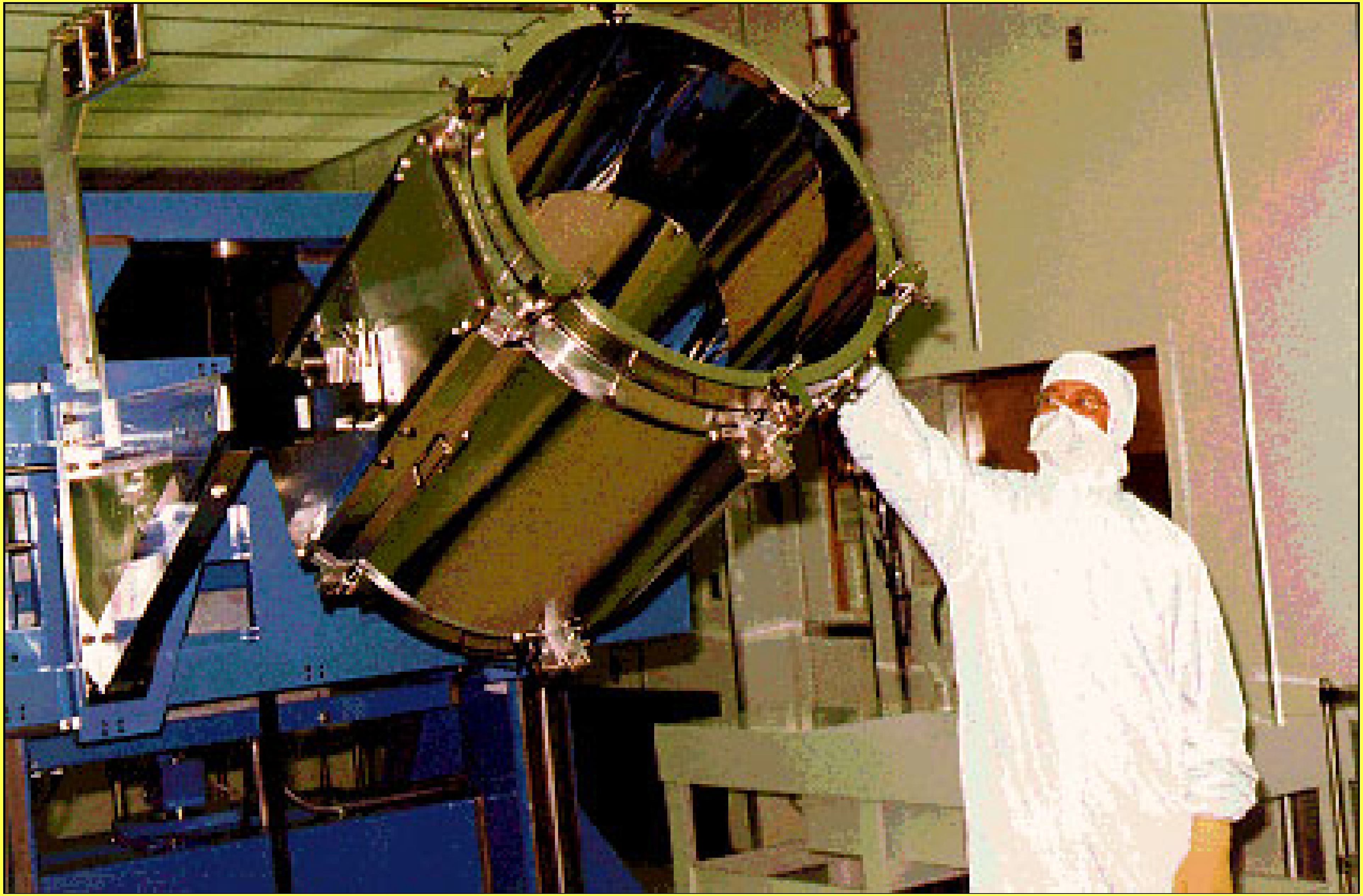
Notice asymmetric effects, due to alignment.

Raytrace calculation.

Chandra: Mirror Polishing



Coated Mirror



Chandra: Mirror Assembly

The Future of X-ray Telescopes

To achieve larger collecting area:

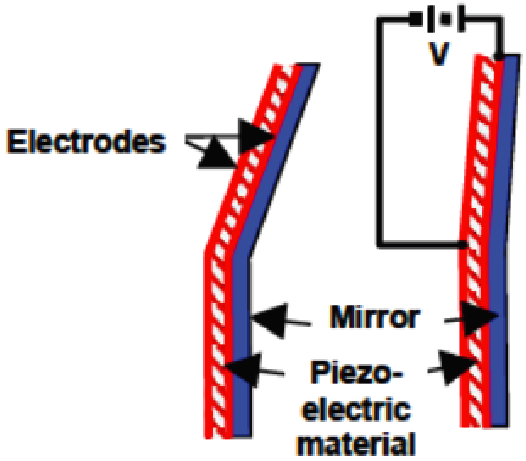
- **Need light weight, thin glass**
- **Large number of densely packed shells**

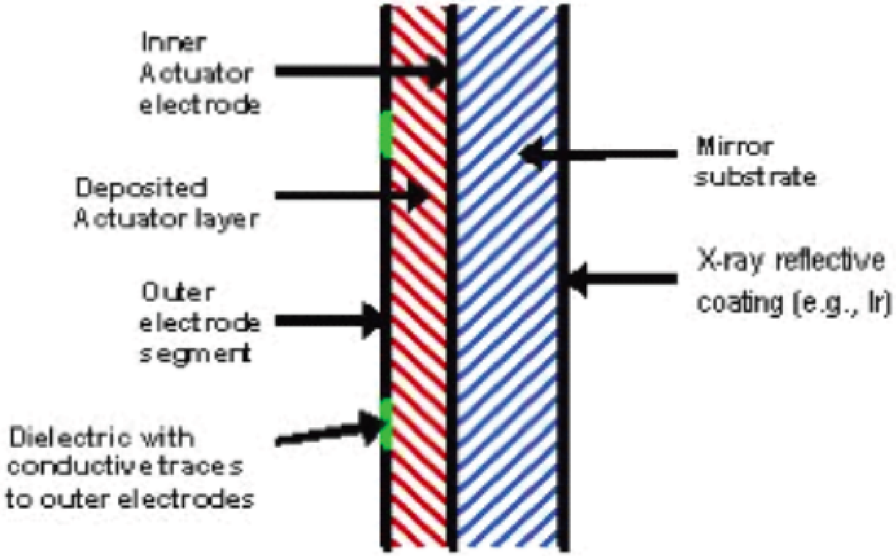
To achieve *Chandra*-like angular resolution

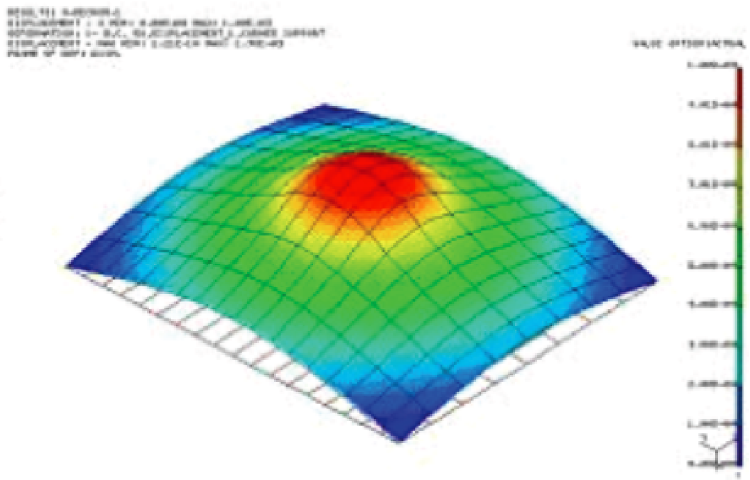
- **Make mirror adjustable, correct on-orbit for gravity release**
- **Thin piezoelectric film deposited on back, controls bi-morph strain**
- **Deposit electrode pattern on piezo layer to create cells**

Bimorph Mirror Concept

SAO





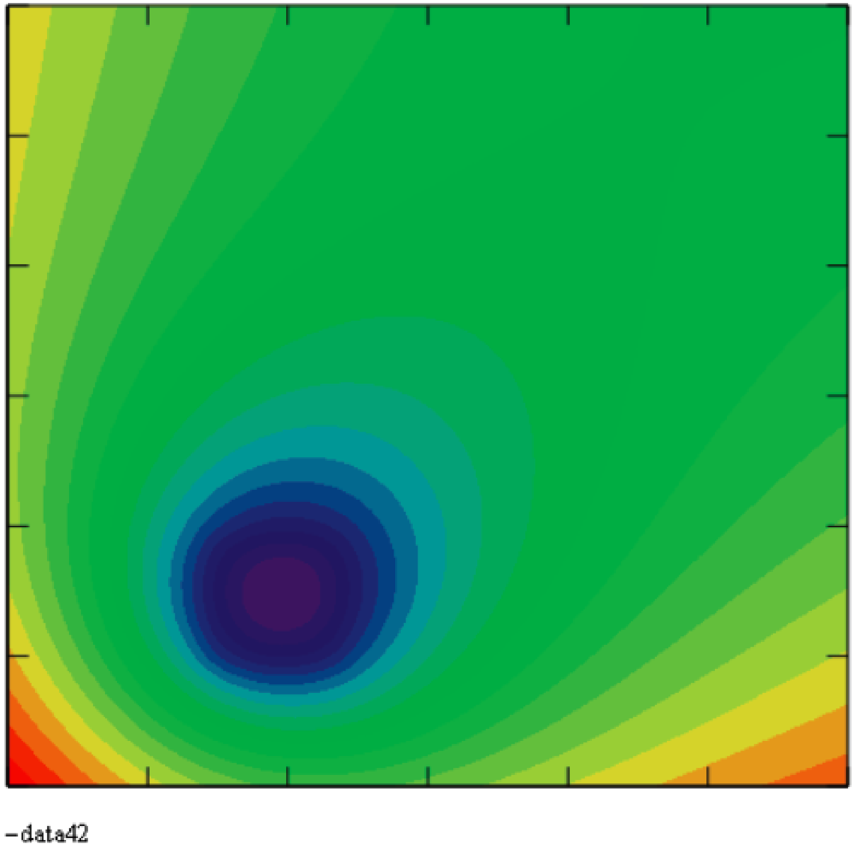


Measurement of Influence Functions (cont'd)

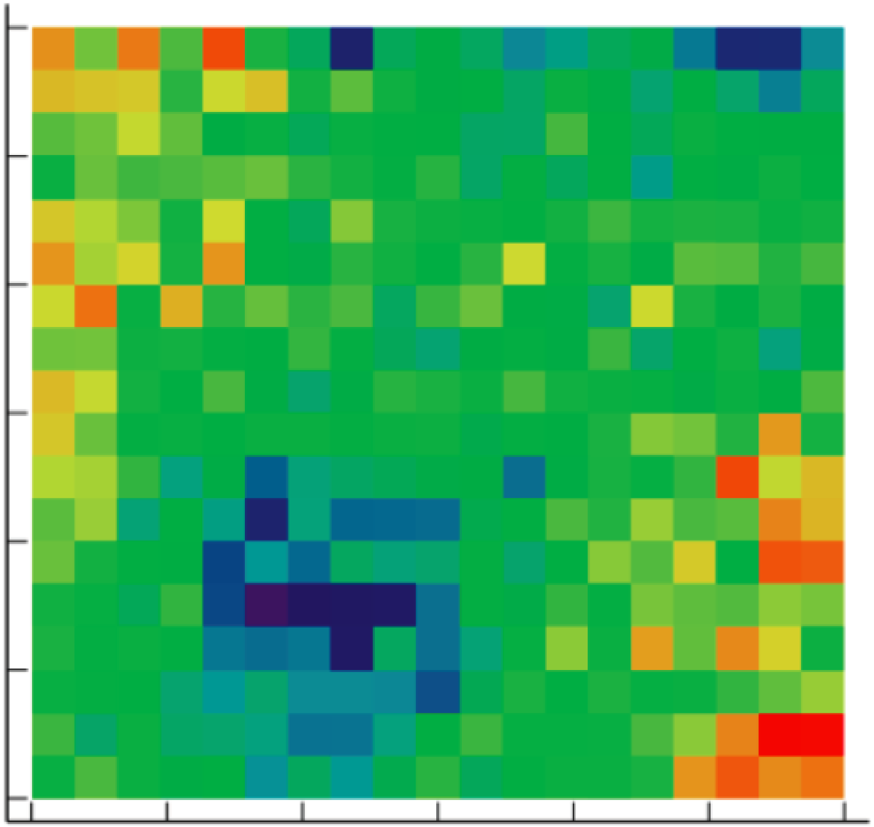
SAO

Normalized modeled and measured influence functions

Single piezo cell energized



FE modeled



Measured

Related Topics

Related Topics:

1. **Aberrations and manufacturing tolerances**
2. **Contamination Control**
3. **Normal Incidence Mirrors**
4. **Multi-layer Coatings**
5. **Gratings**
6. **Refractive Optics**
7. **Polarization**