The Magnetothermal Instability and its Application to Clusters of Galaxies

Ian Parrish

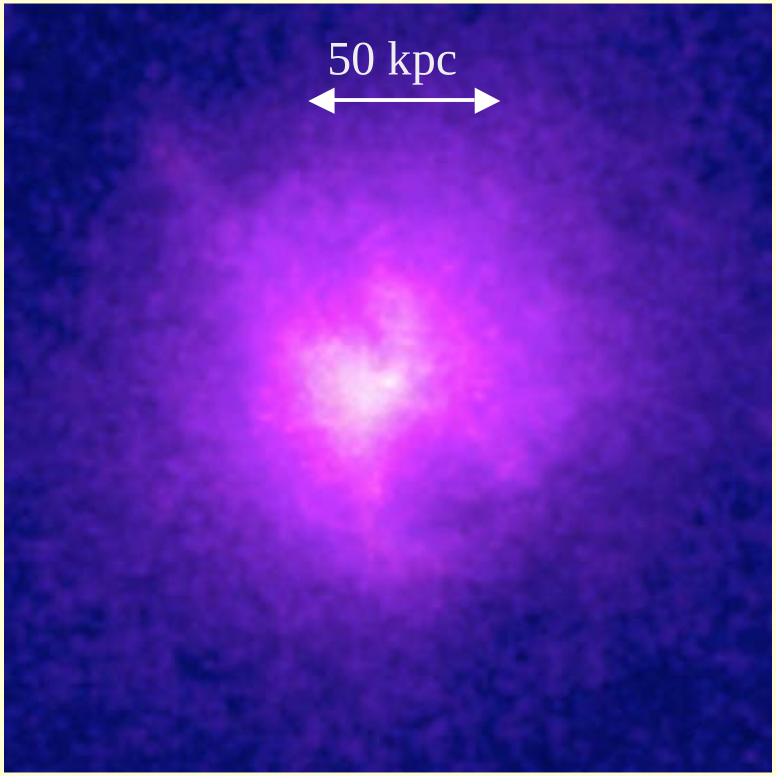
Advisor: James Stone

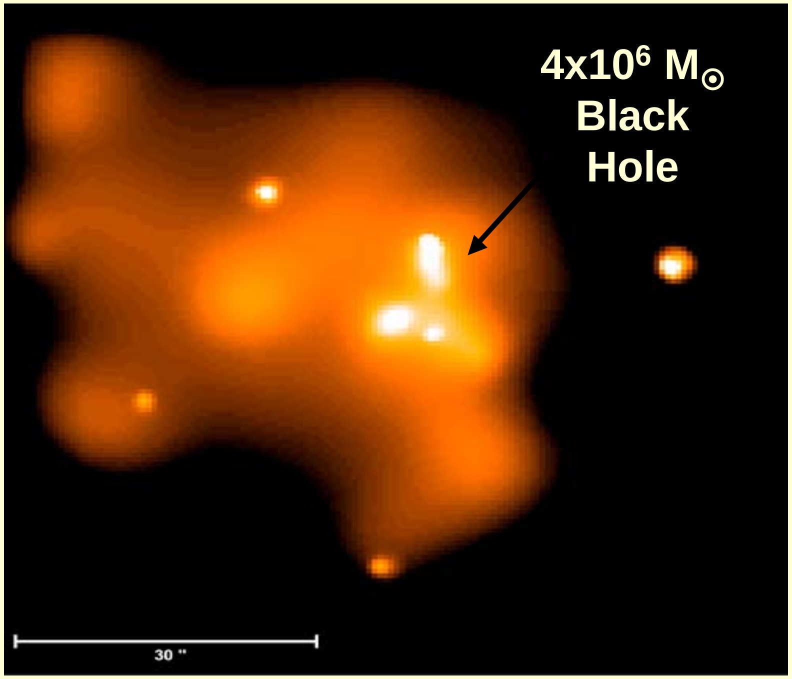
Dept. of Astrophysical Sciences

Princeton University/ UC Berkeley

October 10, 2007

Motivation





Hydra A Cluster (Chandra)

T ~ 4.5 keV   n ~ 10^{-3}-10^{-4}

$\lambda_{mfp} \sim 0.05 R_V >> \rho$

Collisionless Transport

Sgr A*

$T \sim 1$ keV, $n \sim 10$ cm^{-3}

$R_S \sim 10^{12}$ cm

$\lambda_{mfp} \sim 10^{17}$ cm $>> R_S$

Motivating example suggested by E. Quataert

Talk Outline

•**<u>Idea</u>**:

Stability, Instability, and "Backward" Transport in Stratified Fluids, Steve Balbus, 2000.

Physics of the Magnetothermal Instability (MTI).

•**<u>Algorithm:</u>**

Athena: State of the art, massively parallel MHD solver.

Anisotropic thermal conduction module.

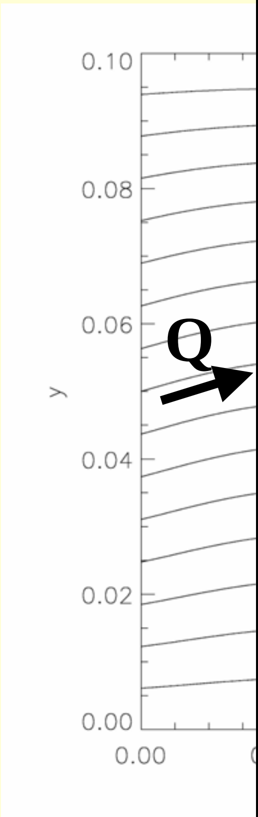
•**<u>Verification and Exploration</u>**

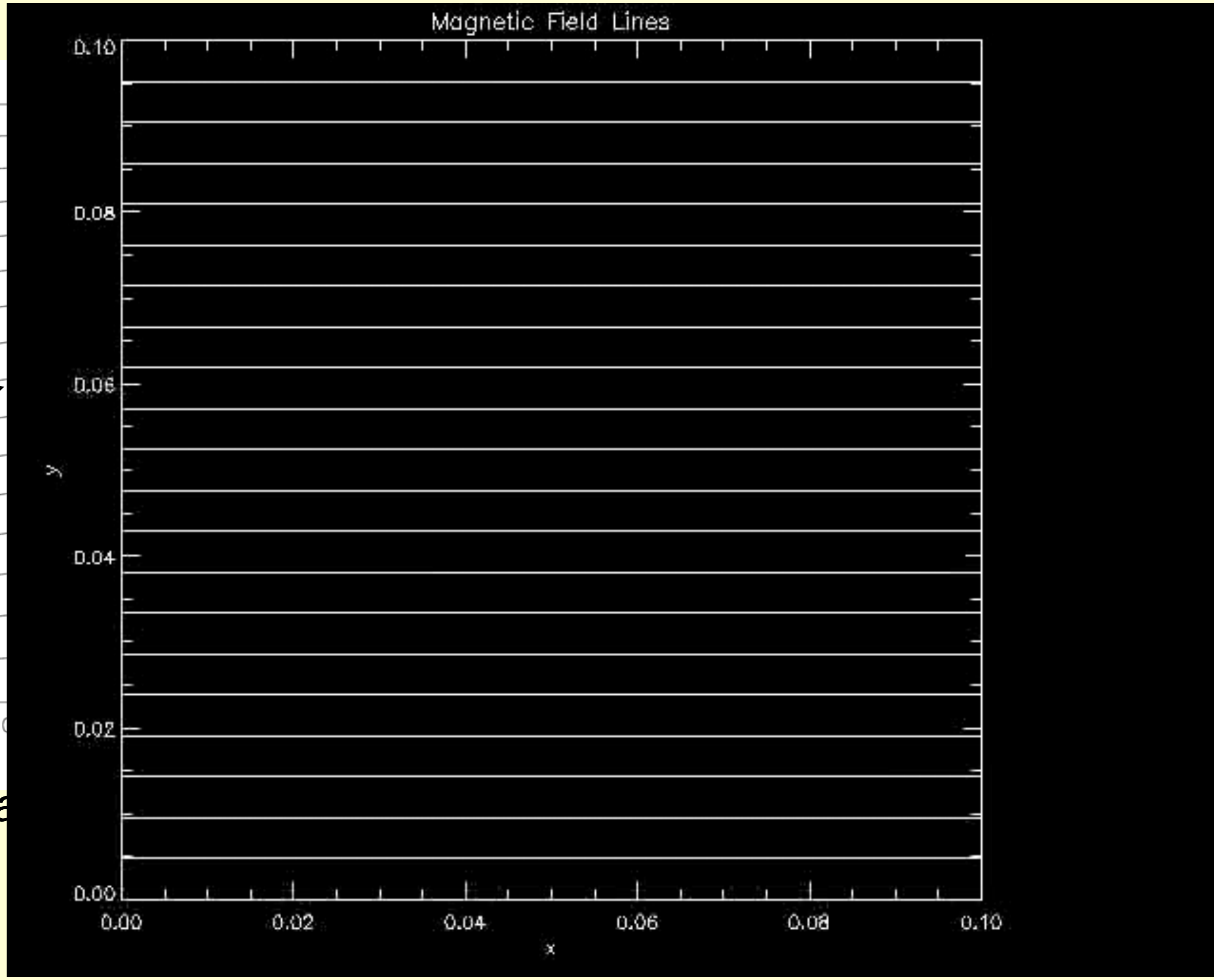
Verification of linear growth rates.

Exploration of nonlinear consequences.

•**Application to Galaxy Clusters**

Idea: Magnetothermal Instability Qualitative Mechanism





Ma

x given
ctivity.

y in a
eld

l Criterion

terion

rion!

Algorithm: MHD with Athena

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

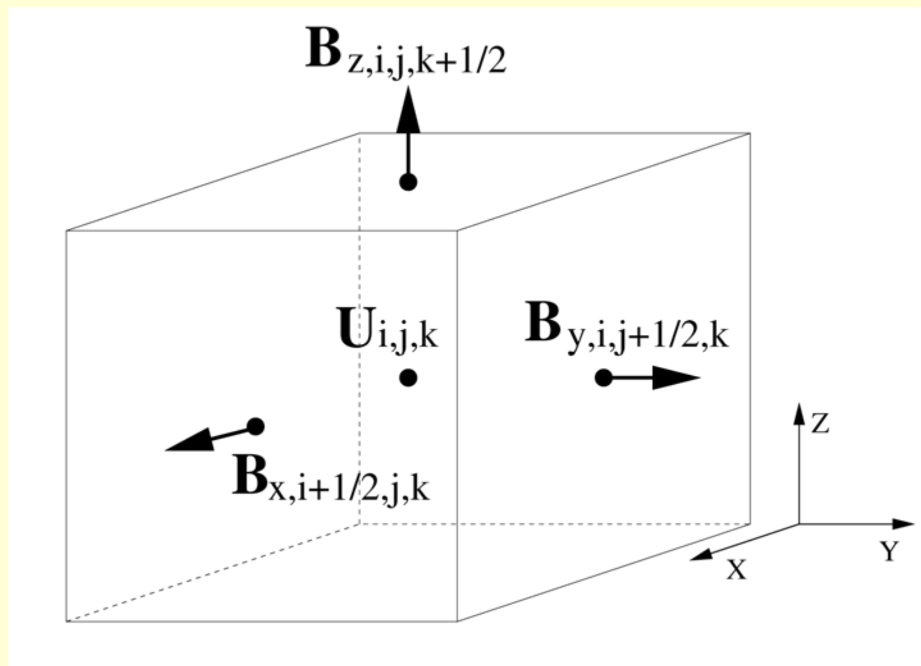
$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + \left(p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{4\pi} \right] = \rho \mathbf{g}, \quad (2)$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \left[\mathbf{v} \left(E + p + \frac{B^2}{8\pi} \right) - \frac{\mathbf{B}(\mathbf{B} \cdot \mathbf{v})}{4\pi} \right] = \rho \mathbf{g} \cdot \mathbf{v} - \nabla \cdot \mathbf{Q}, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) = 0. \quad (4)$$

Athena: Higher order Godunov Scheme

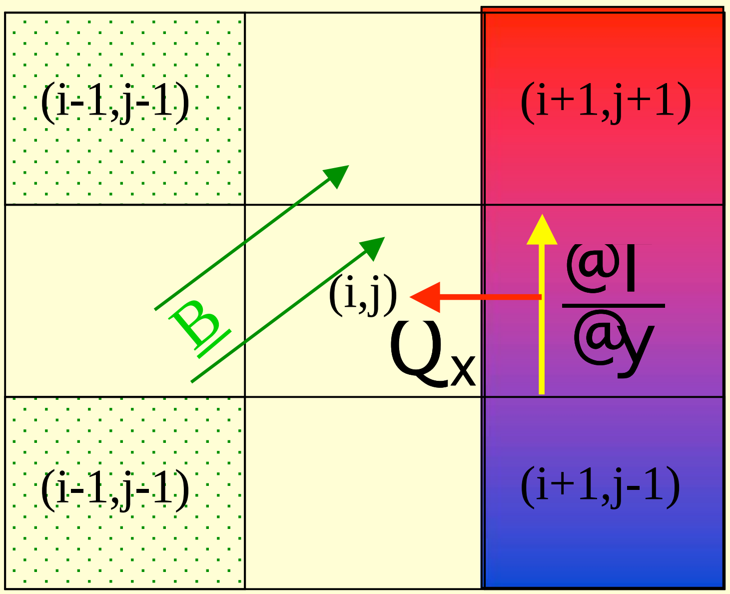
- Constrained Transport for preserving divergence free.
- Unsplit CTU integrator



Algorithm: Heat Conduction

$$\frac{3}{2} P \frac{d \ln P\rho^{-5/3}}{dt} = -\nabla\cdot Q = \nabla\cdot\left(\hat{b}\,\chi_C\,\hat{b}\cdot\nabla T\right)$$

$$Q_x = -\chi_C\left(\hat{b}_x^2 \frac{\partial T}{\partial x} + \hat{b}_x \hat{b}_y \frac{\partial T}{\partial y}\right)$$



Verification

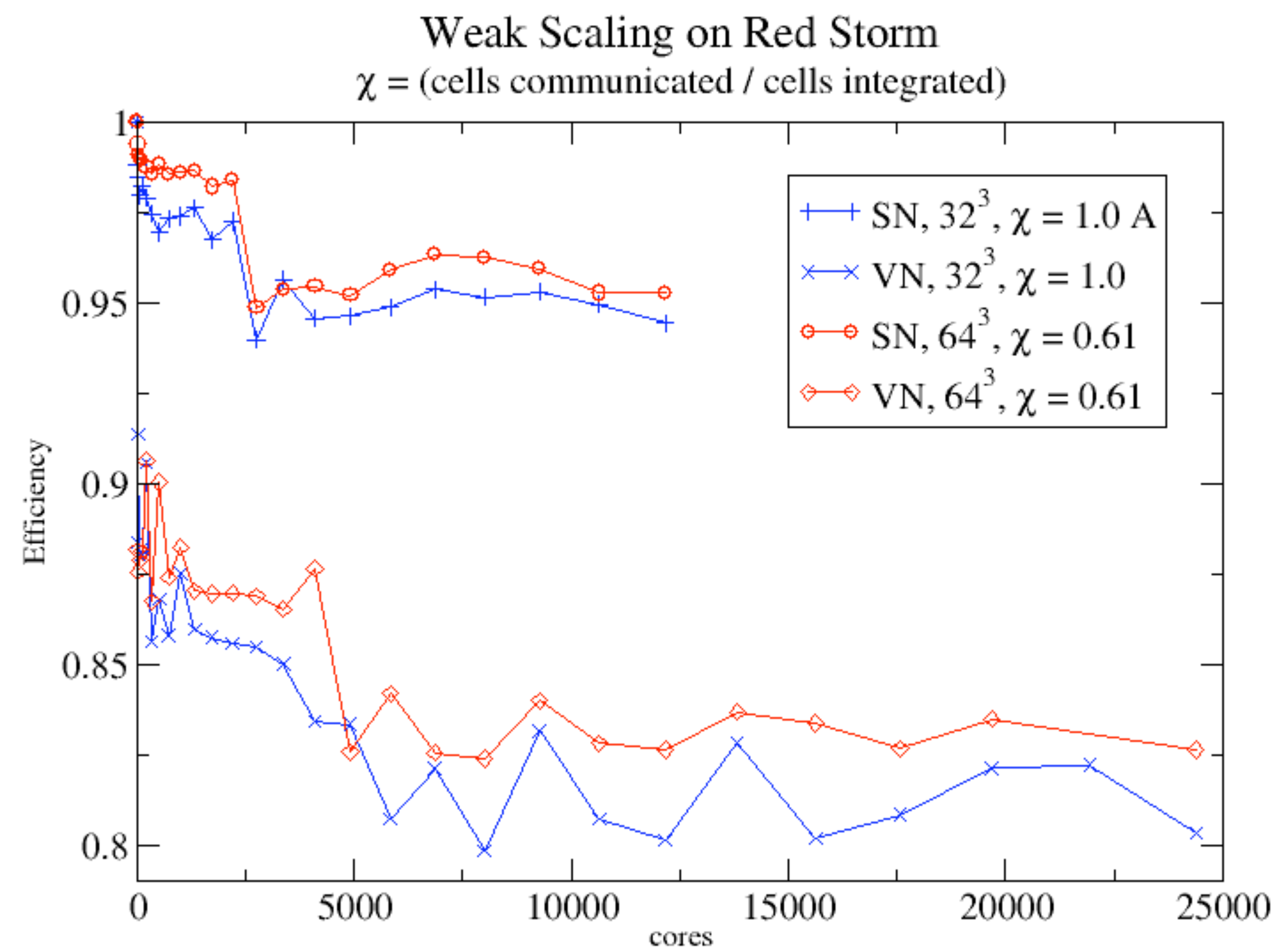
•Gaussian Diffusion: 2nd order accurate.

•Circular Field Lines.

$$\frac{\chi_\perp}{\chi_\parallel} \le 10^{-4}$$

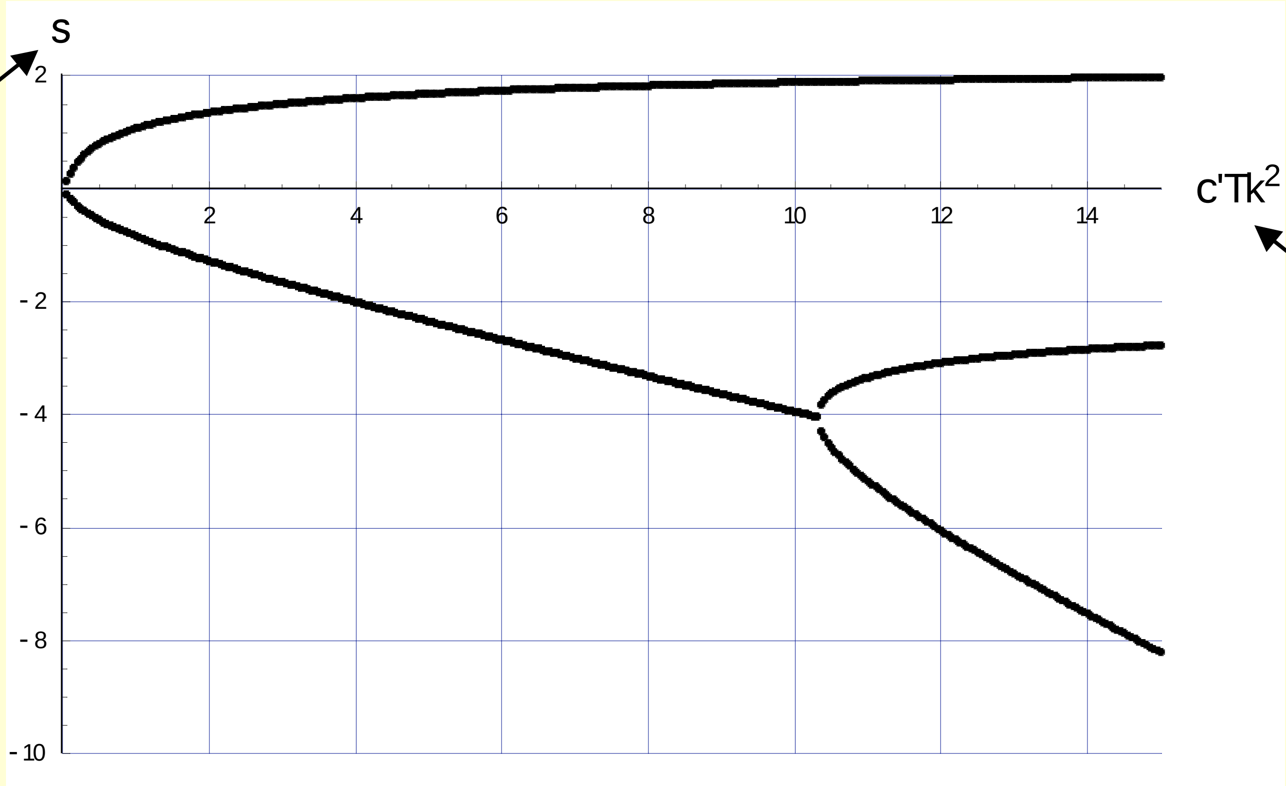
Implemented through sub-cycling diffusion routine.

Algorithm: Performance



Dispersion Relation

Dimensionless Growth Rate



Dimensionless Wavenumber

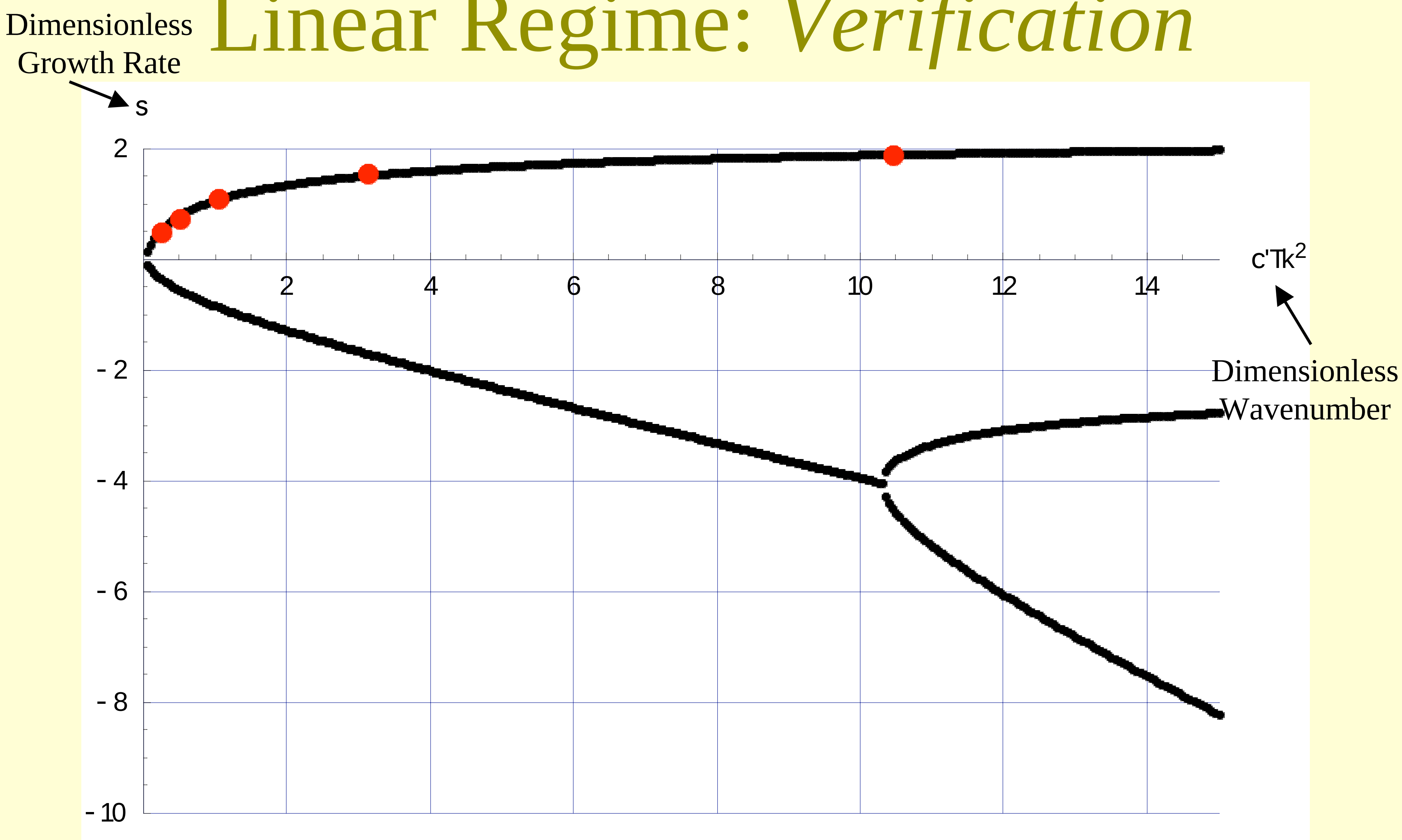
Weak Field Limit

$$\sigma^3 + \frac{\sigma^2}{\gamma}\chi' T k^2 + N^2\sigma - \frac{k^2}{\gamma}\frac{\chi_C'}{\rho}\frac{\partial P}{\partial z}\frac{\partial T}{\partial z} = 0$$

Instability Criterion

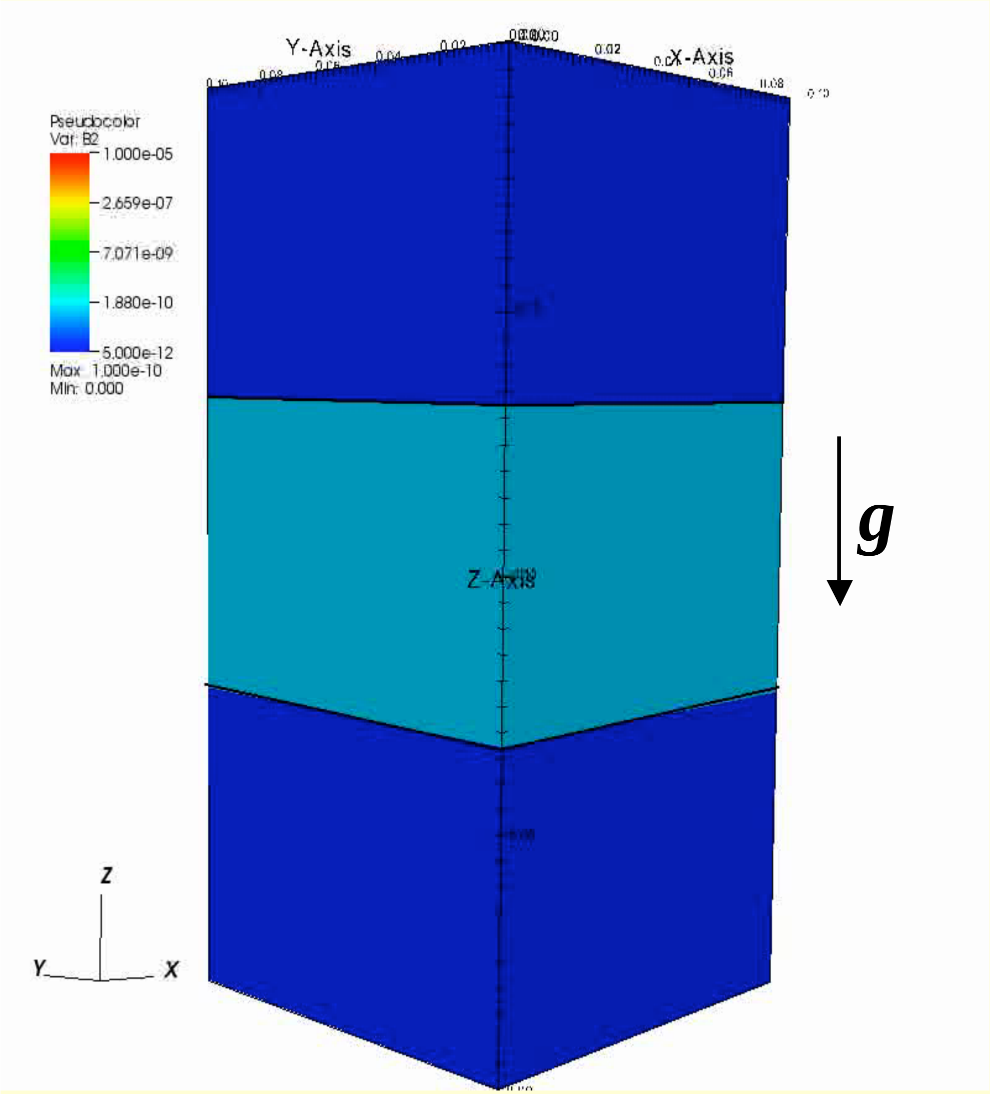
$$k^2 v_A^2 - \frac{\chi_C'}{\rho\chi'}\frac{\partial P}{\partial z}\frac{\partial \ln T}{\partial z} < 0 \xrightarrow{\lim k\to 0} \frac{\partial P}{\partial z}\frac{\partial \ln T}{\partial z} > 0$$

Linear Regime: *Verification*



~3% error

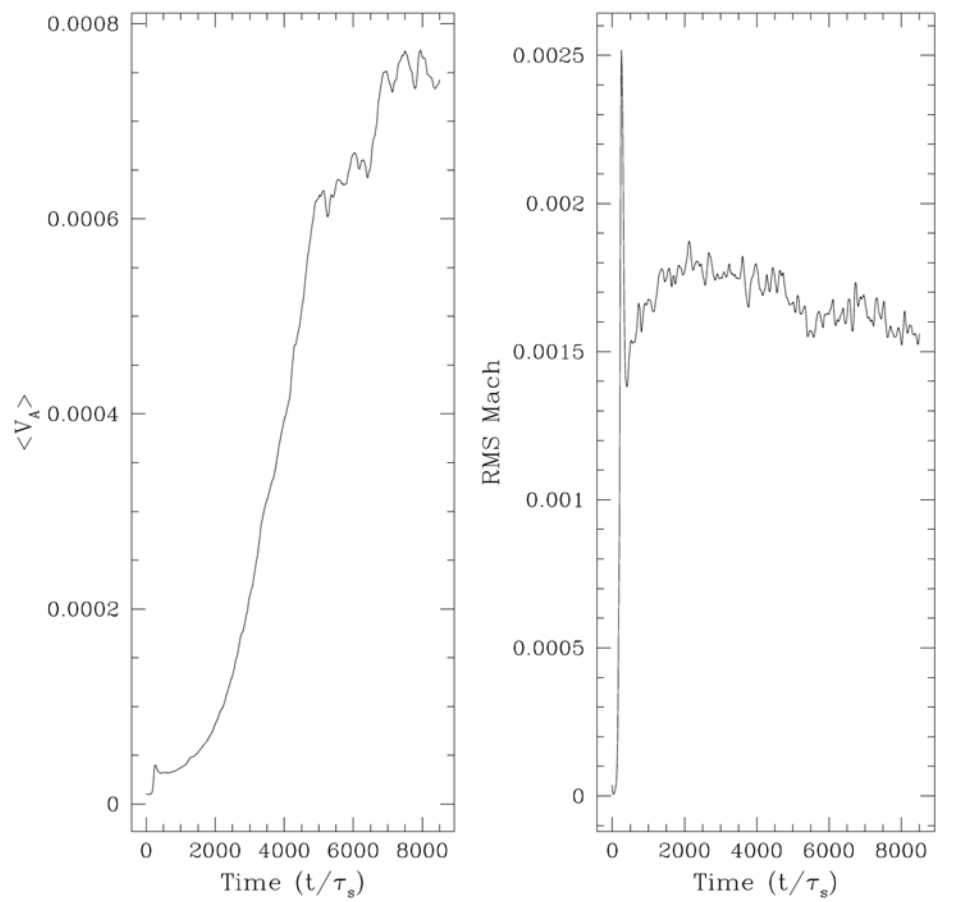
Exploration: 3D Nonlinear Behavior



Magnetic Energy Density = $B^2/2$

- Subsonic convective turbulence, Mach ~ 1.5 x 10^{-3}.
- Magnetic dynamo leads to equipartition with kinetic energy.
- Efficient heat conduction. Steady state heat flux is 1/3 to 1/2 of Spitzer value.

Exploration: 3D Nonlinear Behavior

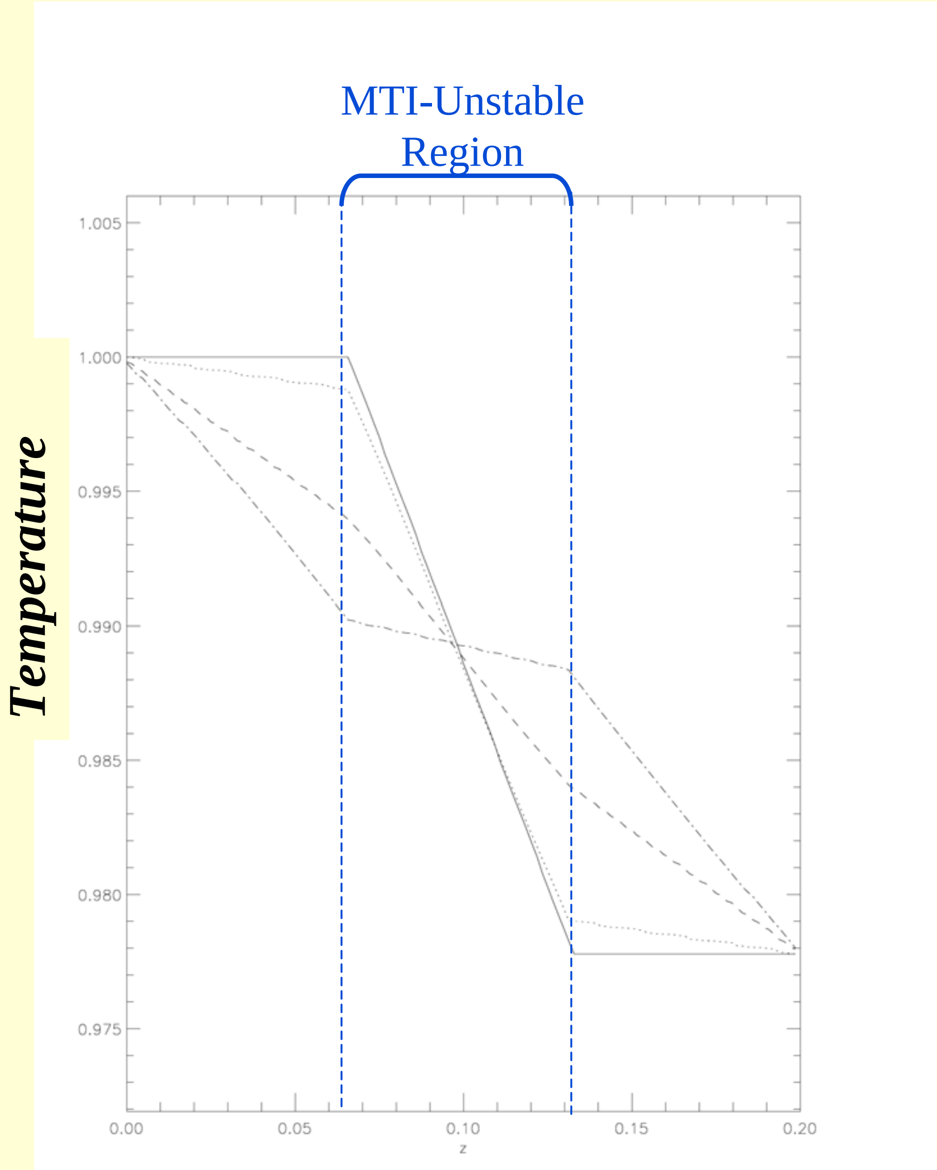


$$V_A^2 = \frac{B^2}{4\pi\rho}$$

RMS Mach

- **Subsonic convective turbulence, Mach ~ 1.5 x 10^{-3}.**
- **Magnetic dynamo leads to equipartition with kinetic energy.**
- Efficient heat conduction. Steady state heat flux is 1/3 to 1/2 of Spitzer value.

Exploration: 3D Nonlinear Behavior



•Subsonic convective turbulence, Mach ~ 1.5 x 10^{-3}.

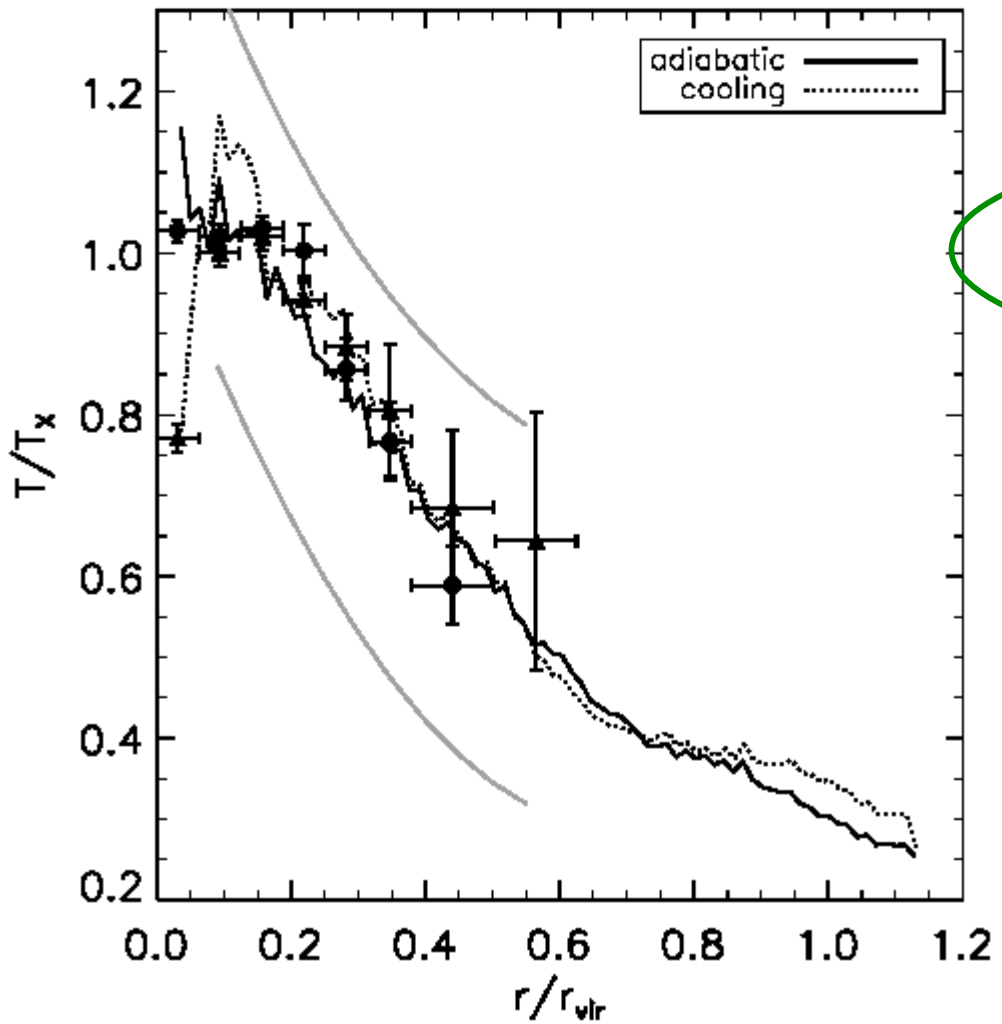
•Magnetic dynamo leads to equipartition with kinetic energy.

•Efficient heat conduction. Steady state heat flux is 1/3 to 1/2 of Spitzer value.

•Temperature profile can be suppressed significantly.

Application: Clusters of Galaxies

Expectations from Structure Formation



Loken, Norman, et al (2002)

Hydro Simulation:
ΛCDM Cosmology, Eulerian

Expect: steep temperature profile

R_v ~ 1-3 Mpc

M ~ $10^{14} - 10^{15}$ solar masses
(84% dark matter, 13% ICM, 3% stars)

T ~ 1-15 keV

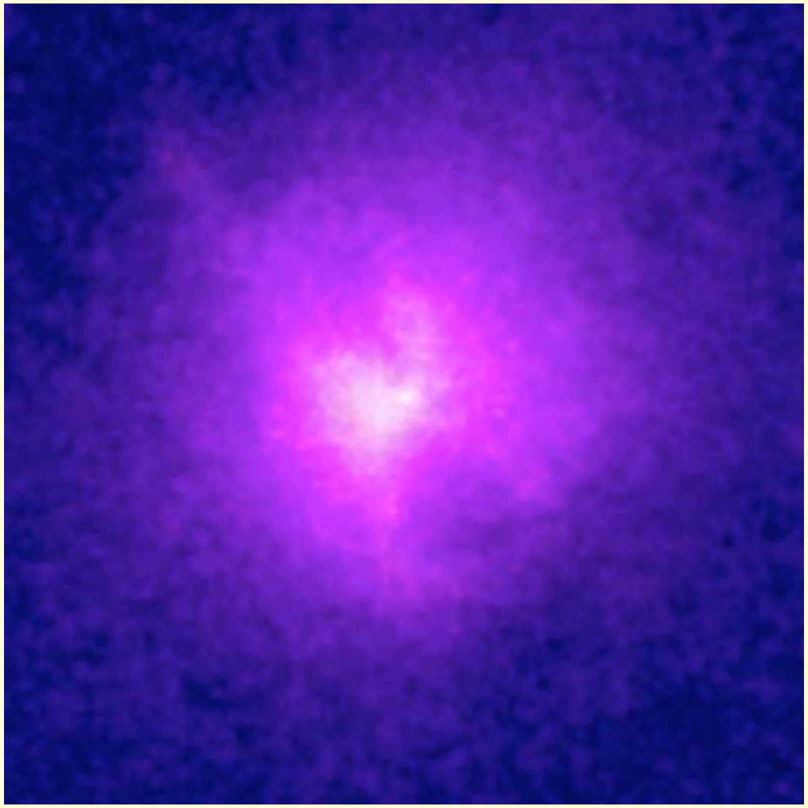
L_X ~ $10^{43} - 10^{46}$ erg/s

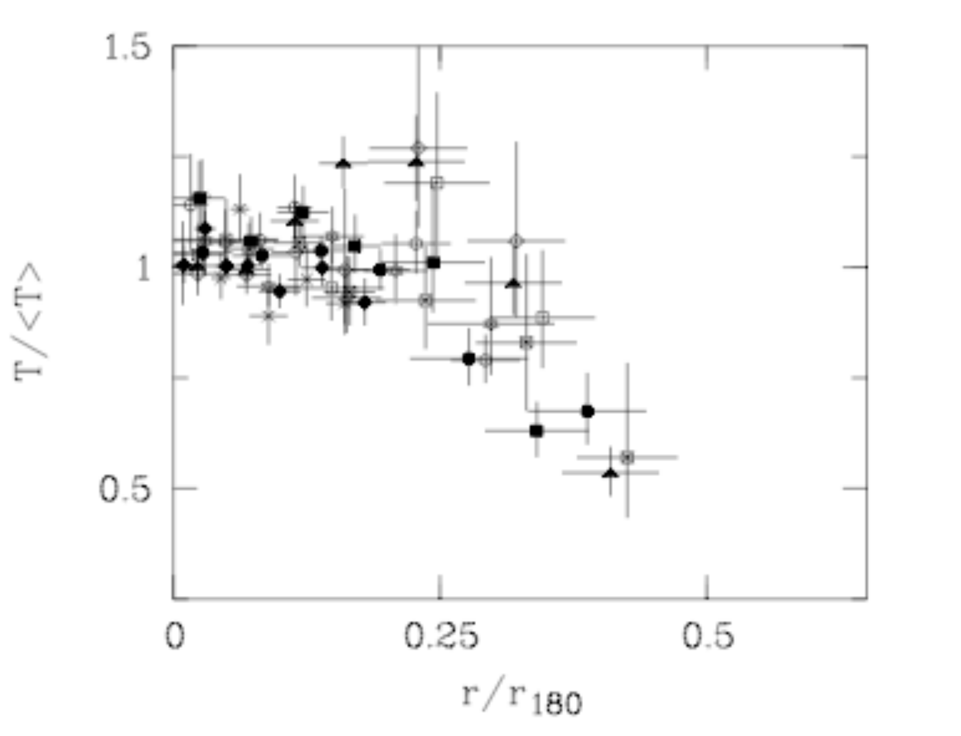
B ~ 1.0 μG

Anisotropic Thermal Conduction Dominates

Application: Clusters of Galaxies

Observational Data



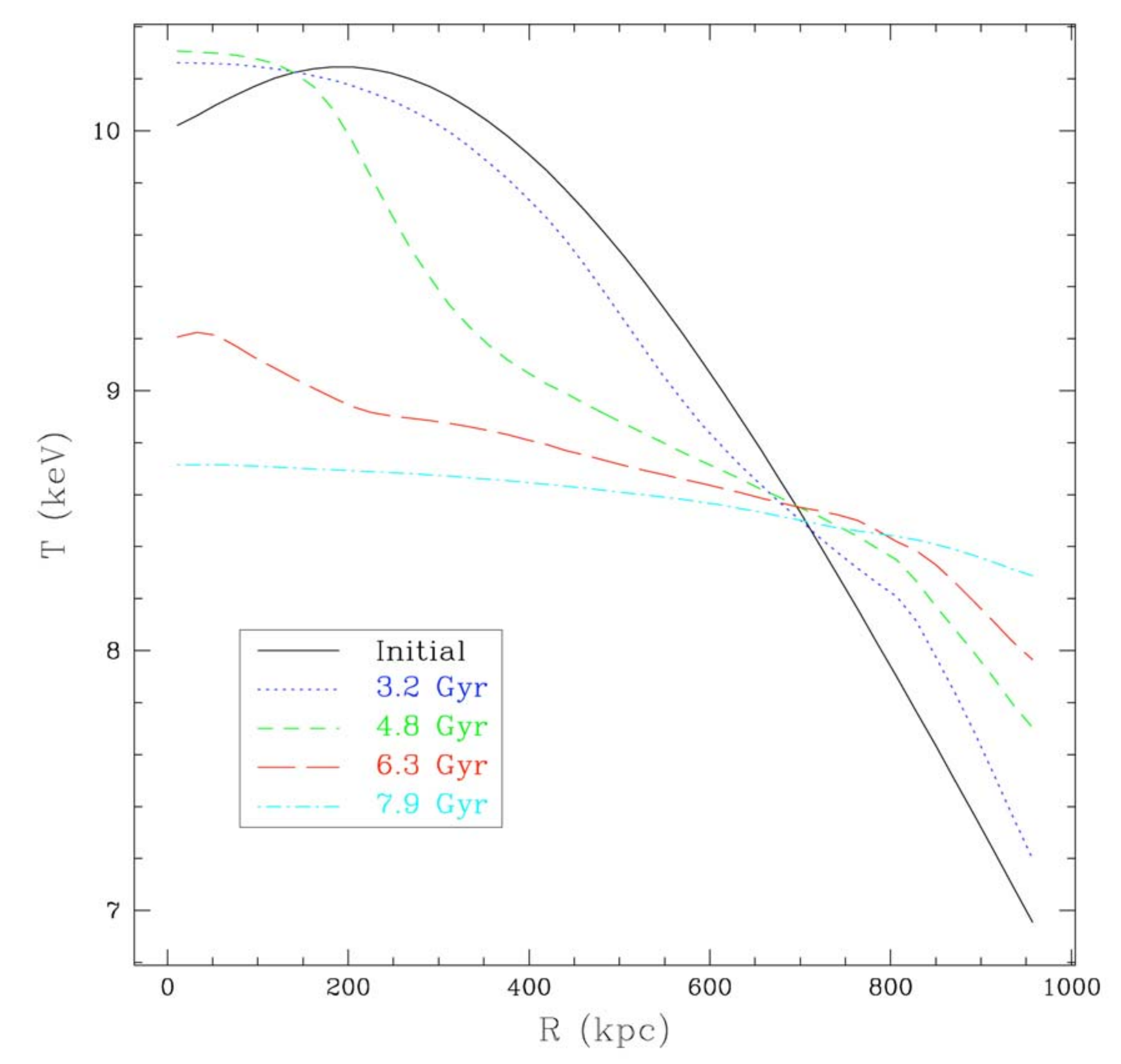


Plot from DeGrandi and Molendi 2002

ICM unstable to the MTI on scales greater than

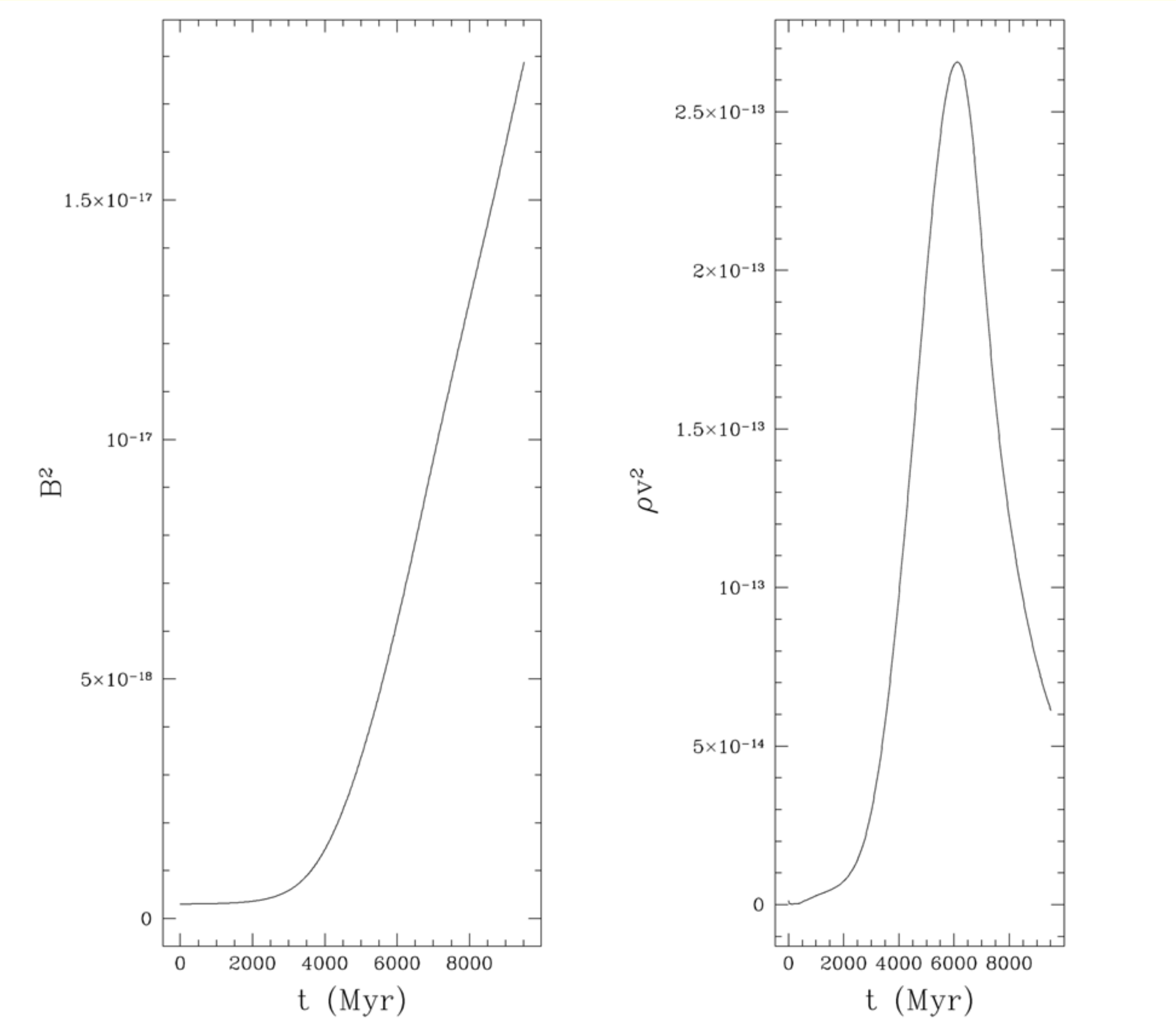
$$\lambda_{\rm crit} = 4.6\,{\rm kpc}\left(\frac{T}{5\,{\rm keV}}\right)^{1/2}\left(\frac{2000}{\beta}\right)^{1/2}$$

Simulation: Clusters of Galaxies



Temperature Profile becomes Isothermal

Simulation: Clusters of Galaxies



Magnetic Dynamo:

B^2 amplified by ~ 60

Vigorous Convection:

Mean Mach: ~ 0.1

Peak Mach: > 0.6

Summary

- Physics of the MTI.
- Verification and validation of MHD + anisotropic thermal conduction.
- Nonlinear behavior of the MTI.
- Application to the thermal structure of clusters of galaxies.

Future Work

- Galaxy cluster heating/cooling mechanisms: jets, bubbles, cosmic rays, etc.
- Application to neutron stars.
- Mergers of galaxy clusters with dark matter.

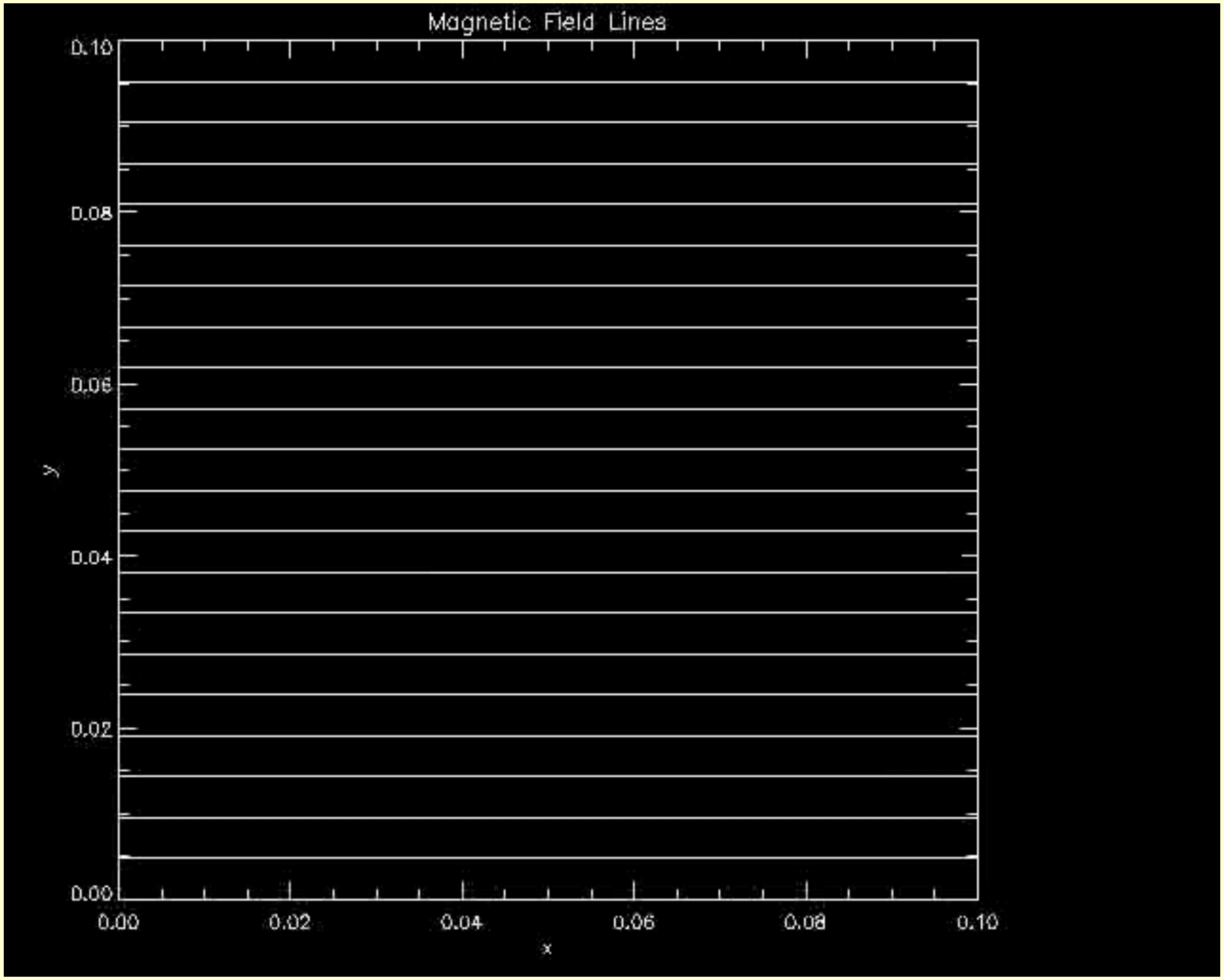
Acknowledgements

- DOE CSGF Fellowship, Chandra Fellowship
- Many calculations performed on Princeton's Orangena Supercomputer

DOE CSGF Department of Energy Computational Science Graduate Fellowship

Questions?

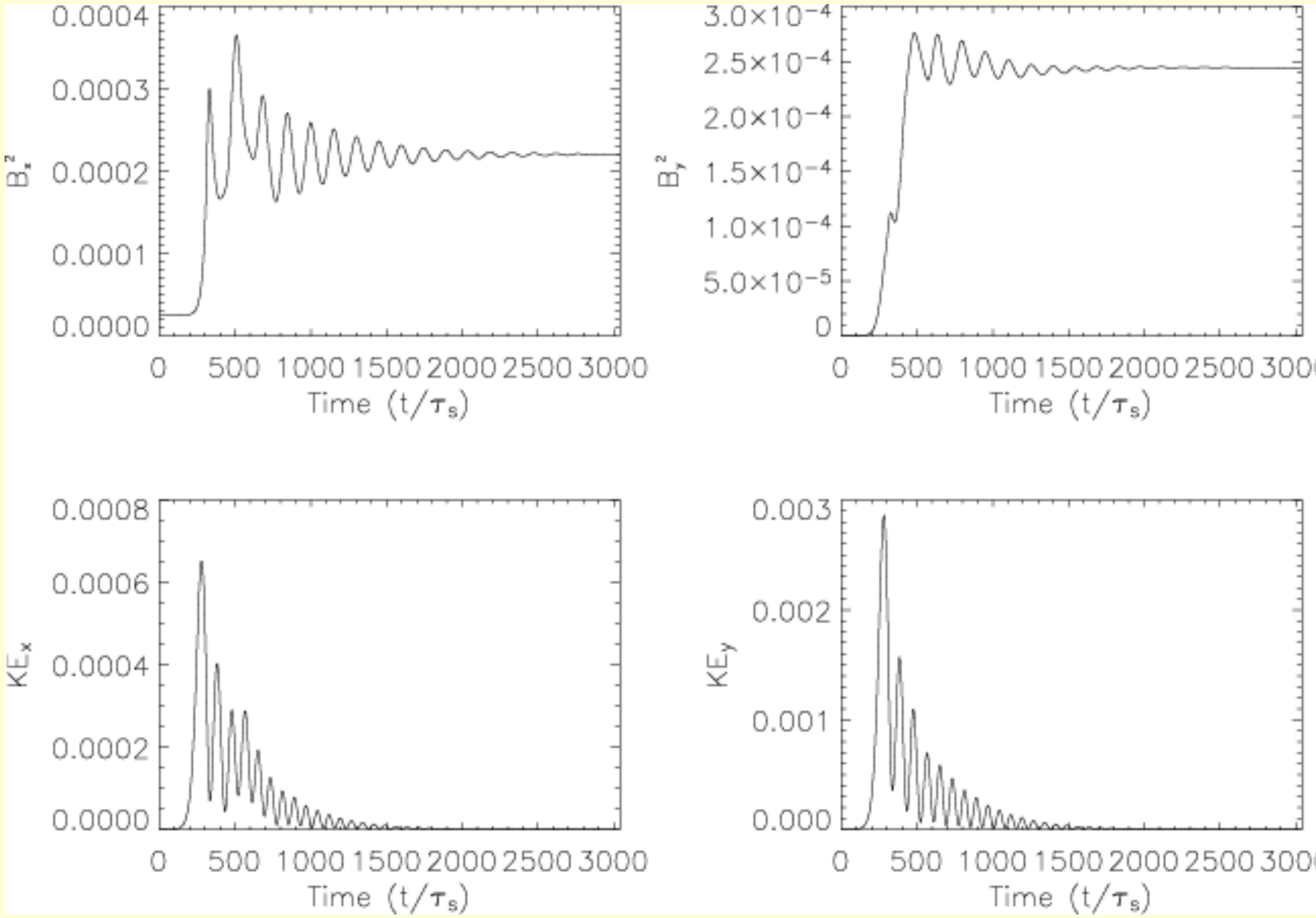
Adiabatic Single Mode Example



Single Mode Evolution

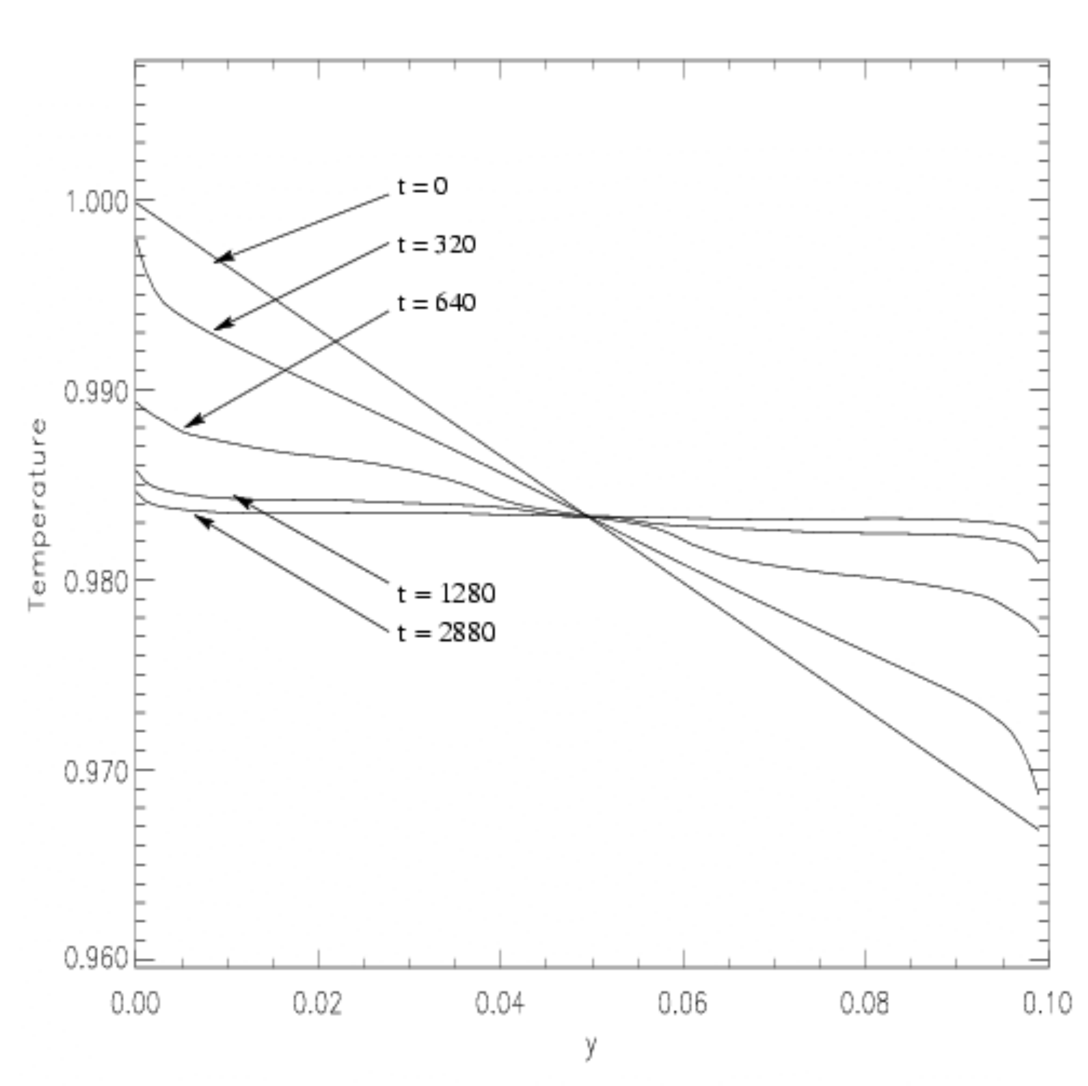
Magnetic Energy Density

Kinetic Energy Density



Single Mode Perturbation $\left(\frac{\chi' T k^2}{N}\right) \approx 1.04, \left(\frac{\sigma}{N}\right) \approx 1.09$

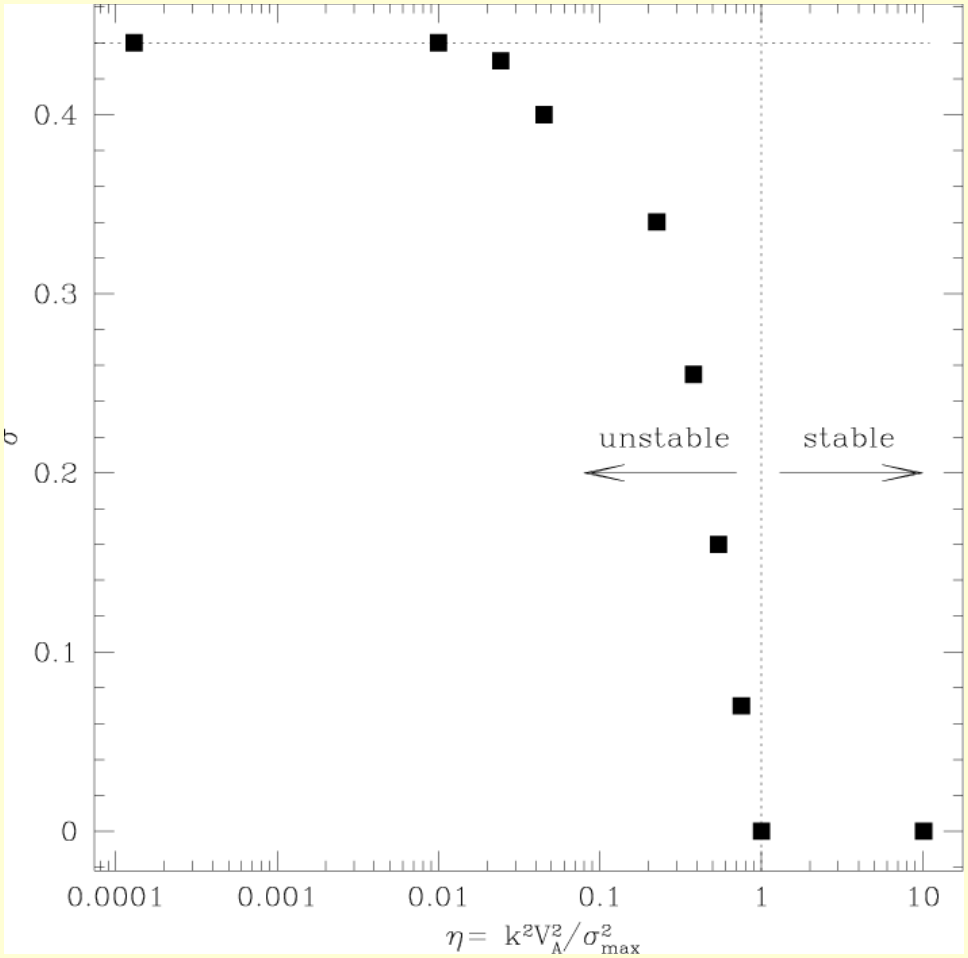
Single Mode Evolution



•Saturated State should be new *isothermal* temperature profile

•Analogous to MRI Saturated State where angular velocity profile is flat.

Dependence on Magnetic Field

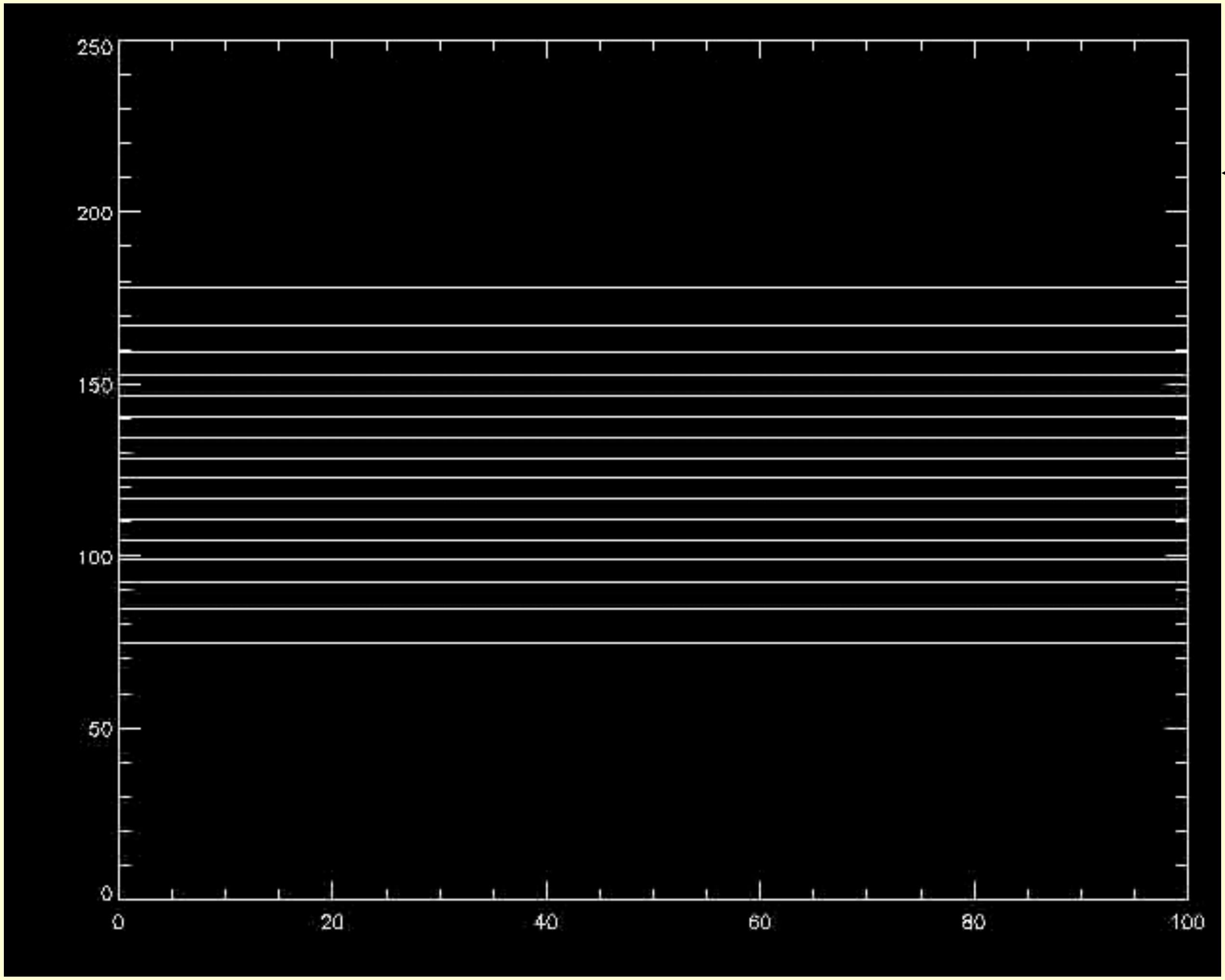


Instability Criterion: $$k^2 v_A^2 - \frac{\chi_C'}{\rho \chi'} \frac{\partial P}{\partial z} \frac{\partial \ln T}{\partial z} < 0$$

Conducting Boundaries

Temperature Fluctuations

Models with Convectively Stable Layers



MTI Stable

MTI Unstable

MTI Stable

•Heat flux primarily due to *Advective* component.

•Very efficient total heat flow

Future Work & Applications

Full 3-D Calculations

- •Potential for a dynamo in three-dimensions (early evidence)
- •Convection is intrinsically 3D

•Application-Specific Simulations

- •Clusters of Galaxies
- •Atmospheres of Neutron Stars

Acknowledgements: Aristotle Socrates, Prateek Sharma, Steve Balbus, Ben Chandran, Elliot Quataert, Nadia Zakamska, Greg Hammett

•Funding: Department of Energy Computational Science Graduate Fellowship (CSGF)

SUPPLEMENTARY MATERIAL

Analogy with MRI

Magneto-Rotational

- Keplerian Profile
- Conserved Quantity:

  Angular Momentum

- Free Energy Source:

  Angular Velocity Gradient

- *Weak Field* Required

Unstable When:

$$k^2 v_A^2 + \frac{d\Omega}{d \ln R} < 0$$

Magneto-Thermal

- Convectively Stable Profile
- Conserved Quantity:

  Entropy

- Free Energy Source:

  Temperature Gradient

- *Weak Field* Required

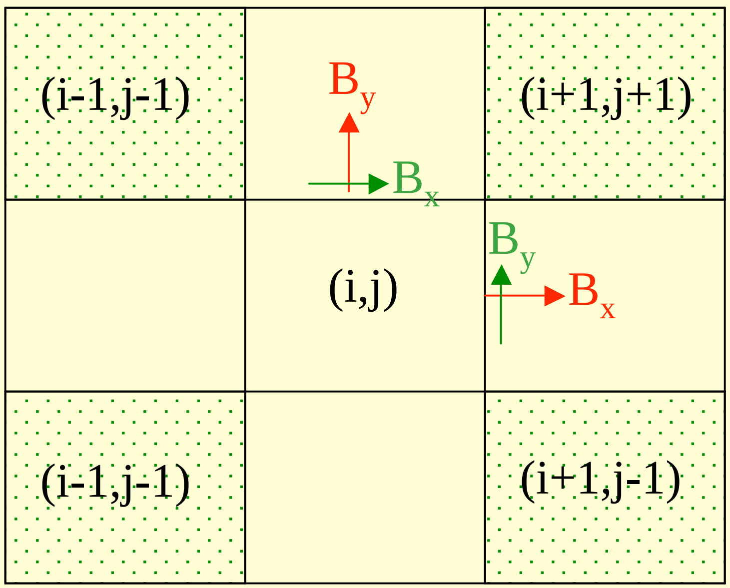
Unstable When:

$$k^2 v_A^2 - \frac{\chi_C'}{\rho \chi'} \frac{\partial P}{\partial z} \frac{\partial \ln T}{\partial z} < 0$$

Heat Conduction Algorithm

$$\frac{3}{2}\frac{Pd\ln P\rho^{-\gamma}}{dt} = -\nabla\cdot\mathbf{Q} = -\nabla\cdot\left[\hat{b}\left(\chi\hat{b}\cdot\nabla T\right)\right]$$

$$\text{R.H.S.} = \chi\left\{\frac{\partial}{\partial x}\left[b_x\left(b_x\frac{\partial T}{\partial x}+b_y\frac{\partial T}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left[b_y\left(b_x\frac{\partial T}{\partial x}+b_y\frac{\partial T}{\partial y}\right)\right]\right\}$$

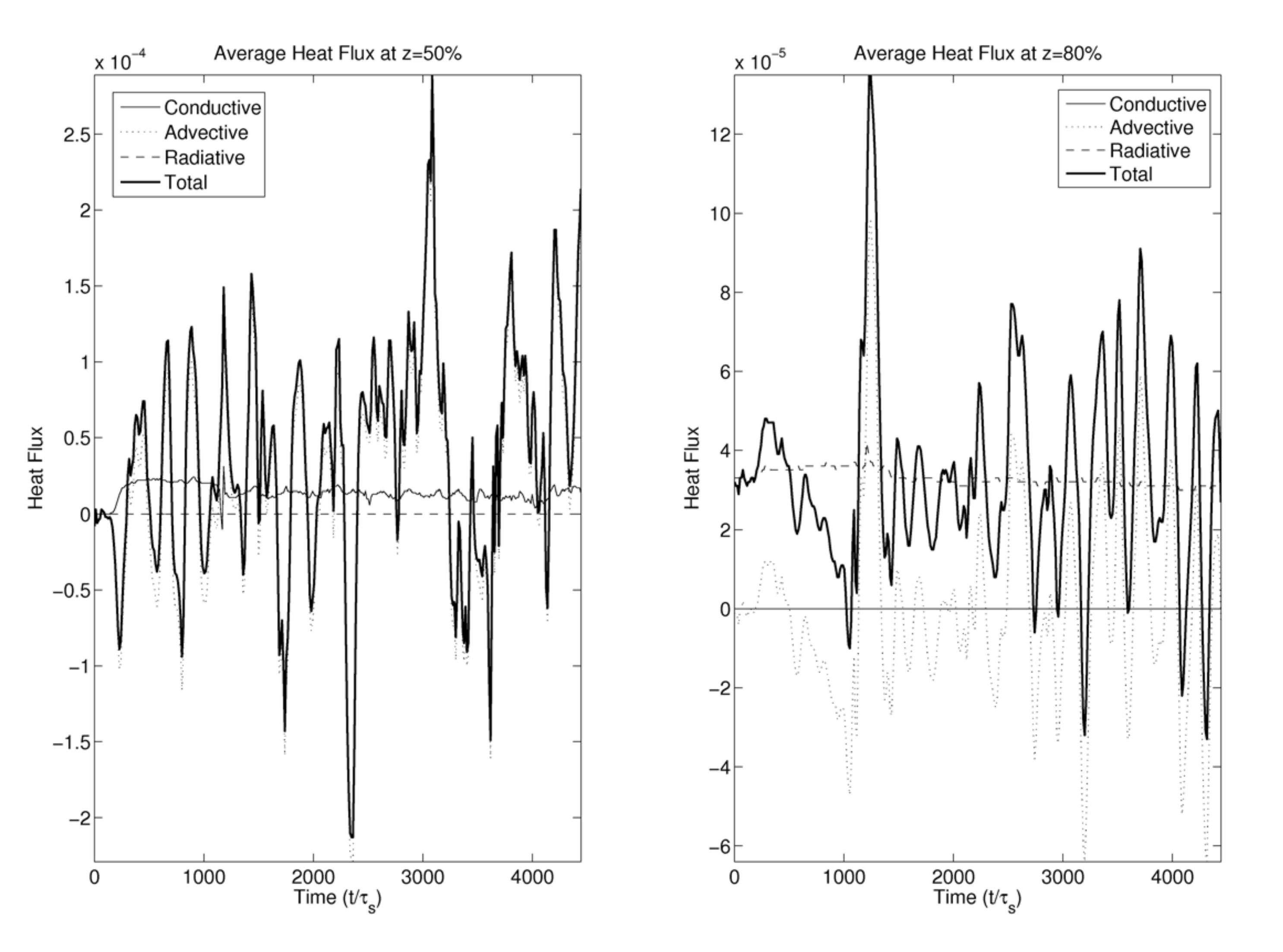


1. Magnetic Fields Defined at Faces
2. Interpolate Fields
3. Calculate Unit Vectors

$$\frac{\partial}{\partial x}\left(\hat{b}_x\hat{b}_y\frac{\partial T}{\partial y}\right) = \frac{\hat{b}_{x,i+1,j}\hat{b}_{y,i+1,j}\left(\frac{T_{i+1,j+1}-T_{i+1,j-1}}{2\Delta y}\right)-\hat{b}_{x,i-1,j}\hat{b}_{y,ii1,j}\left(\frac{T_{i-1,j+1}-T_{i-1,j-1}}{2\Delta y}\right)}{2\Delta x}$$

+Symmetric Term

Heat Flux with Stable Layers



Outline & Motivation

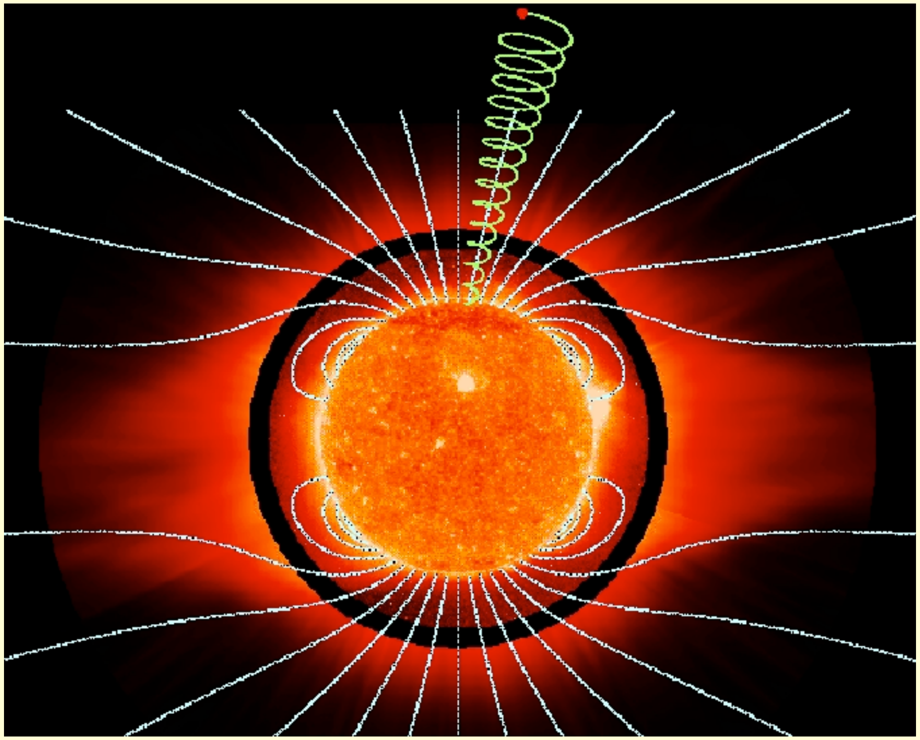
Goal: Numerical simulation of plasma physics with MHD in astrophysics.

- Verification of algorithms
- Application to Astrophysical Problems

Outline:

- Physics of the Magnetothermal Instability (MTI)
- Verification of Growth Rates
- Nonlinear Consequences
- Application to Galaxy Clusters

Solar Corona

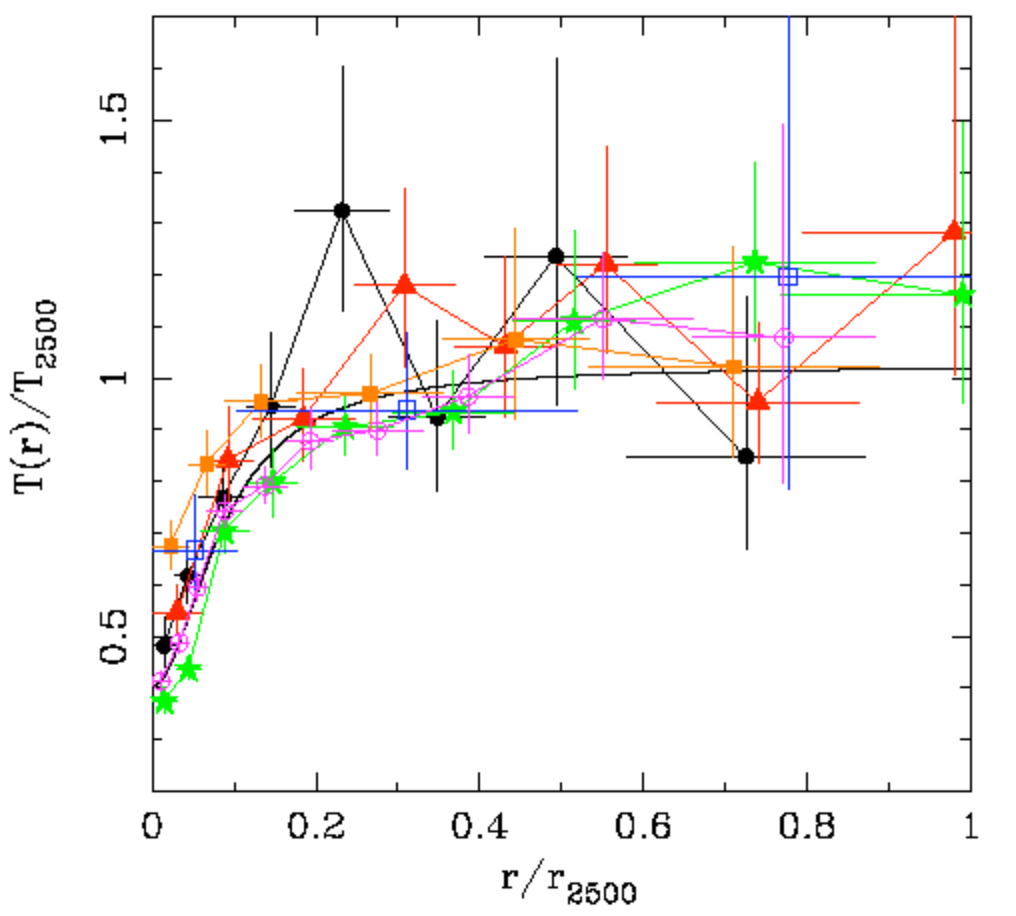


Around 2 $R_{\odot}$:

$n \sim 3 \times 10^{15}$, $T \sim$ few 10^6 K

$\lambda_{mfp} >$ distance from the sun

Cooling Flows?



•*Problem*: Cooling time at center of cluster much faster than age of cluster

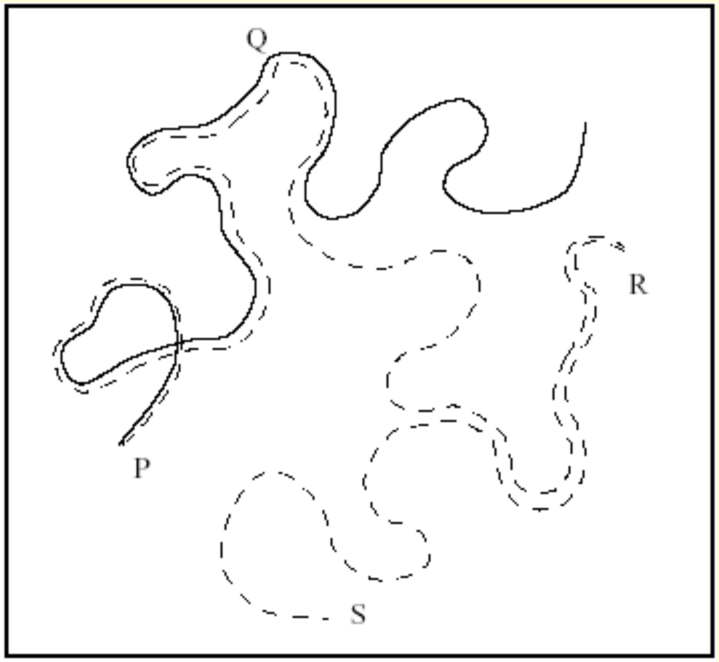
•*Theory A*: Cooling flow drops out of obs. To colder phase

•*Observation*: No cool mass observed!

•*Theory B*: Another source of heat…from a central AGN, from cosmic rays, Compton heating, **thermal conduction**

(Graphic from Peterson & Fabian, astro-ph/0512549)

Thermal Conduction



•Rechester & Rosenbluth/Chandran & Cowley: Effective conductivity for chaotic field lines…Spitzer/100 (too slow)

•Narayan & Medvedev: Consider multiple correlation lengths…Spitzer/a few (fast enough?)

•Zakamska & Narayan: Sometimes it works.

•ZN: AGN heating models produce thermal instability!

•Chandran: Generalization of MTI to include cosmic ray pressure

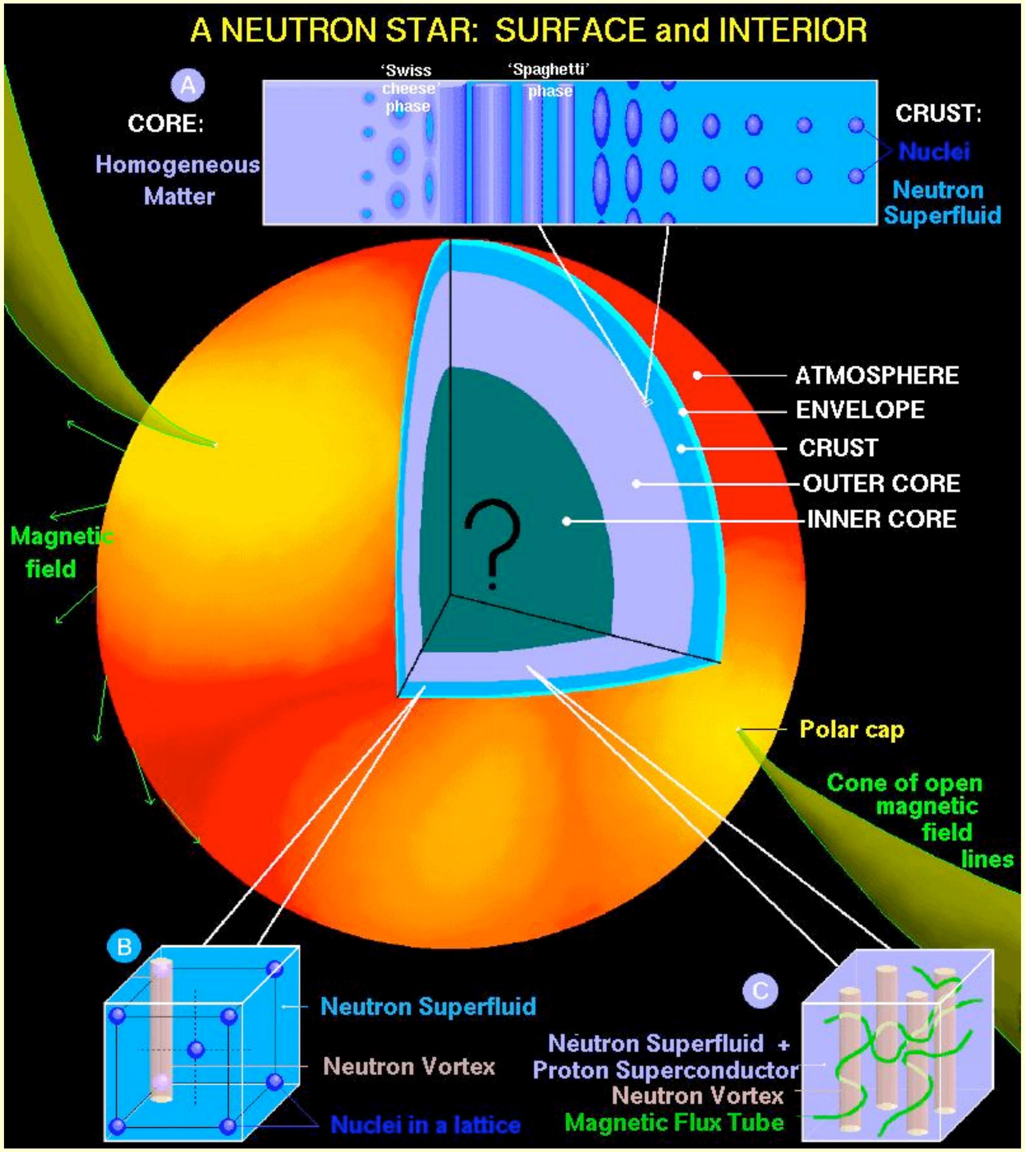
Clusters: Case for Simulation

- Difficult to calculate effective conductivity in tangled field line structure analytically
- Heat transport requires convective mixing length model
- Convection modifies field structure….feedback loop

<u>Plan:</u>

- Softened NFW Potential: $\rho_{DM} = \dfrac{M_0}{(r + r_c)(r + r_s)^2}$
- Initial Hydrostatic Equilibrium: Convectively Stable, MTI Unstable
- Magnetic Field: Smooth Azimuthal/Chaotic
- Resolution: Scales down to 5-10 kpc (the coherence length) within 200 kpc box requires roughly a 512^3 domain

Application II: Neutron Stars



<u>Neutron Star Parameters</u>

•R = 10 km (Manhattan)

•M = 1.4 solar masses

•B = $10^8 - 10^{15}$ G

<u>Properties</u>

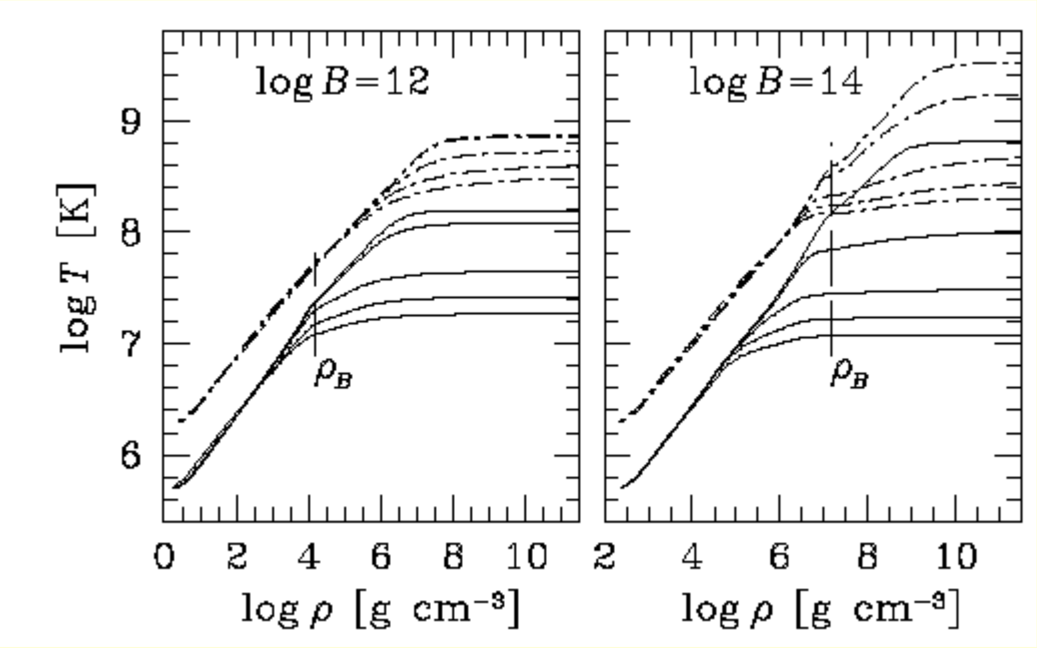
•Semi-relativistic

•Semi-degenerate

•In ocean, not fully quasi-neutral

•In crust, Coulomb Crystal?

Neutron Star Atmosphere



(from Ventura & Potekhin)

T_{out} = 5 x 10^5 K (solid)

T_{out} = 2 x 10^6 K (dashed)

Various values of cosθ

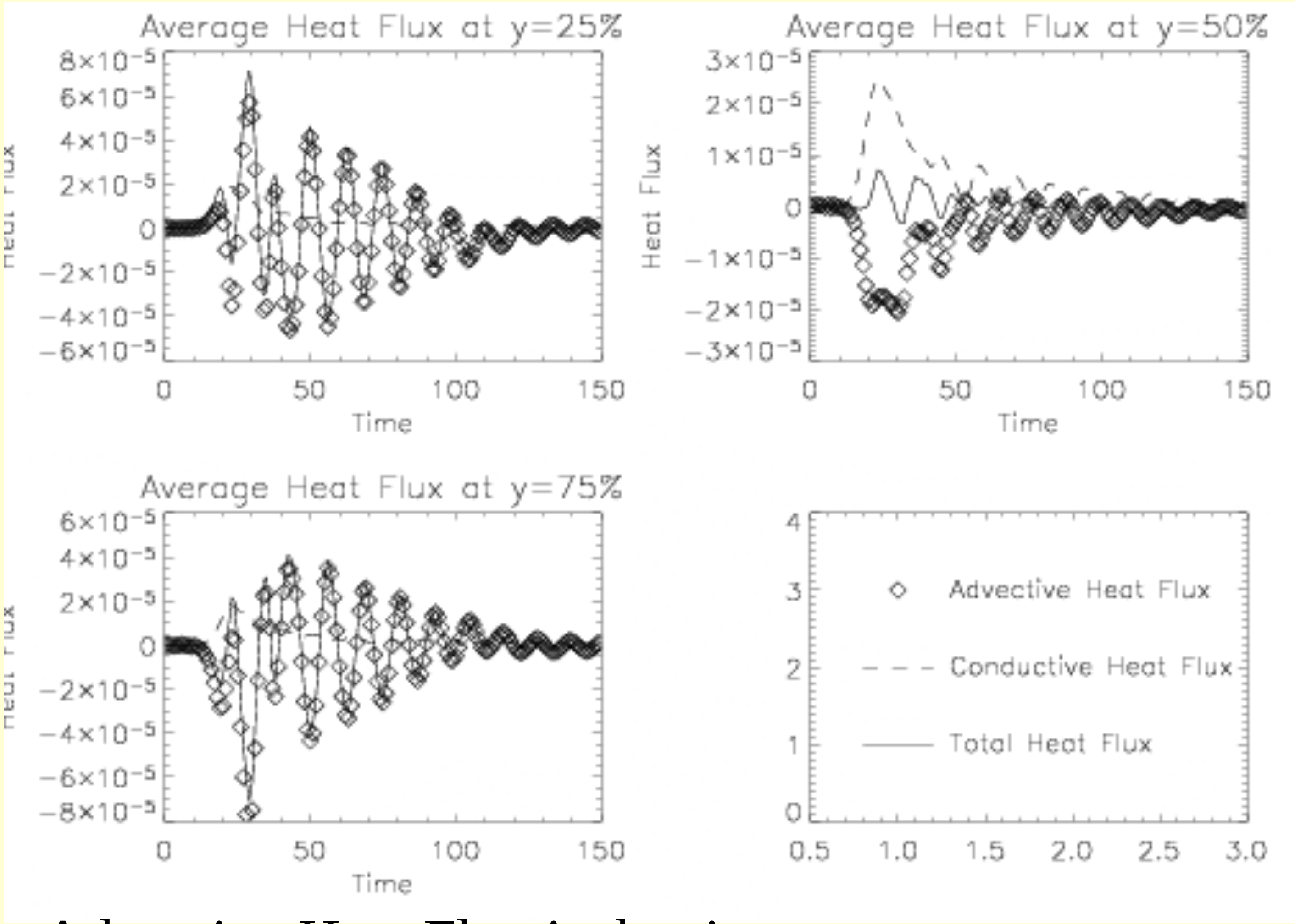
<u>Construct an atmosphere</u>

- EOS: Paczynski, semi-degenerate, semi-relativistic
- Opacity: Thompson scattering (dominant), free-free emission
- Conduction: Degenerate, reduced Debye screening (Schatz, et al)
- Integrate constant-flux atmosphere with shooting method.

Instability Analysis and Simulation

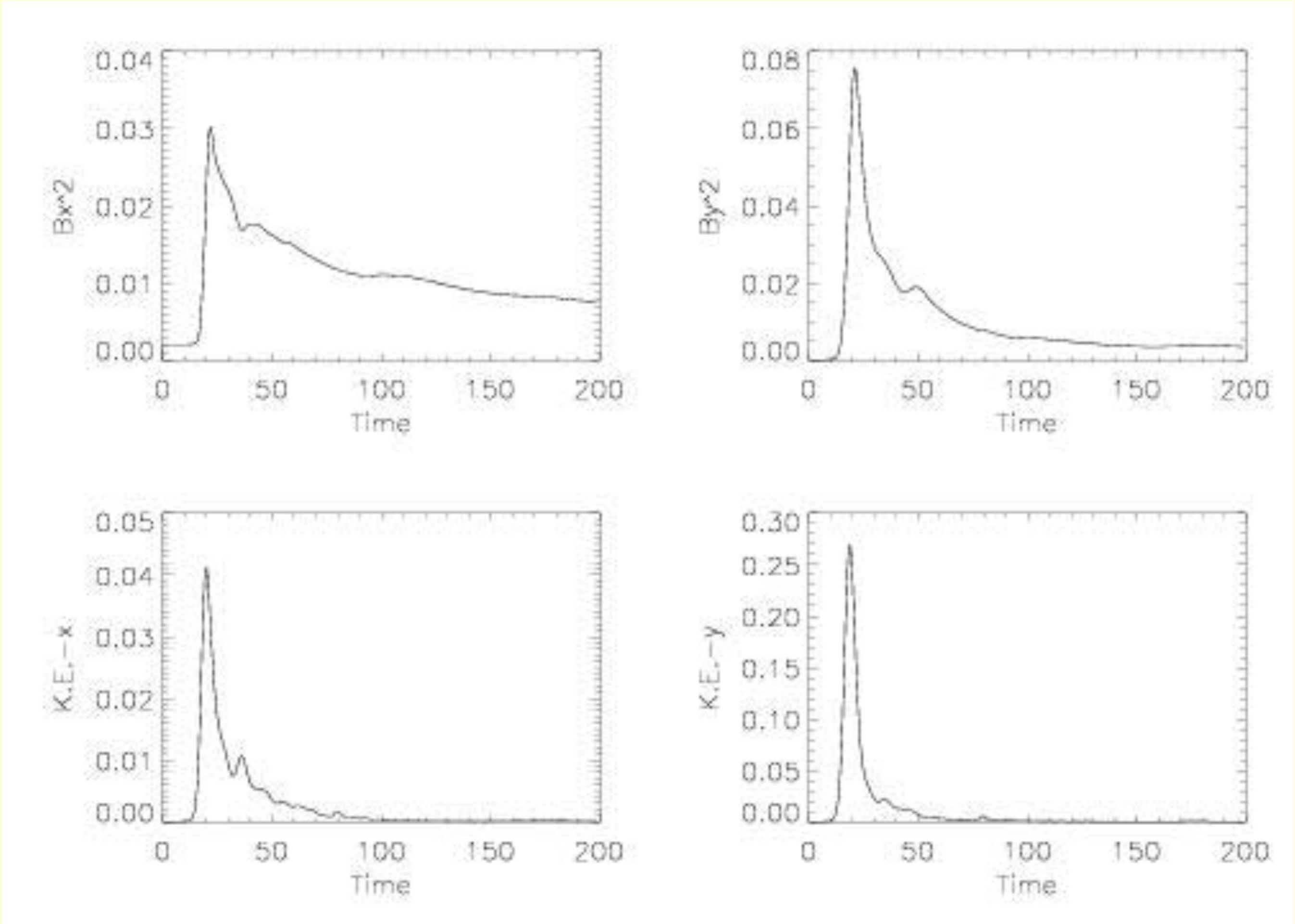
- Potential for instability near equator where B-field lines are perpendicular to temperature gradient
- MTI damped:
  - Outer parts due to radiative transport
  - Inner part due to stronger magnetic field and cross-field collisions
- Check analytically if unstable
- Simulate plane-parallel patch in 3D with Athena
  - Estimate heat transport properties, and new saturated T-profile

Non-Linear Evolution III



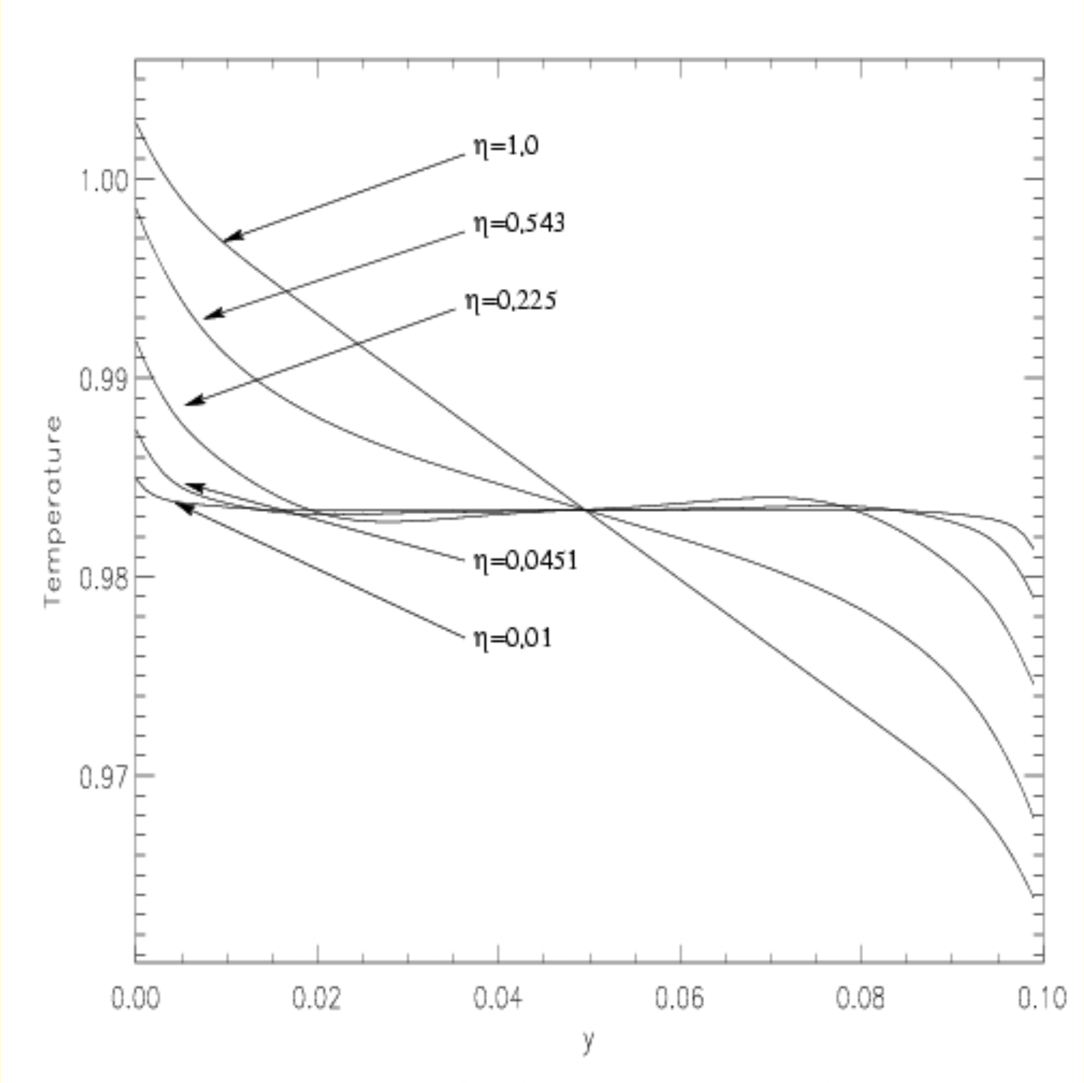
- Advective Heat Flux is dominant
- Settling of atmosphere to isothermal equilibrium

Adiabatic Multimode Evolution



No Net Magnetic Flux leads to decay by Anti-Dynamo Theorem

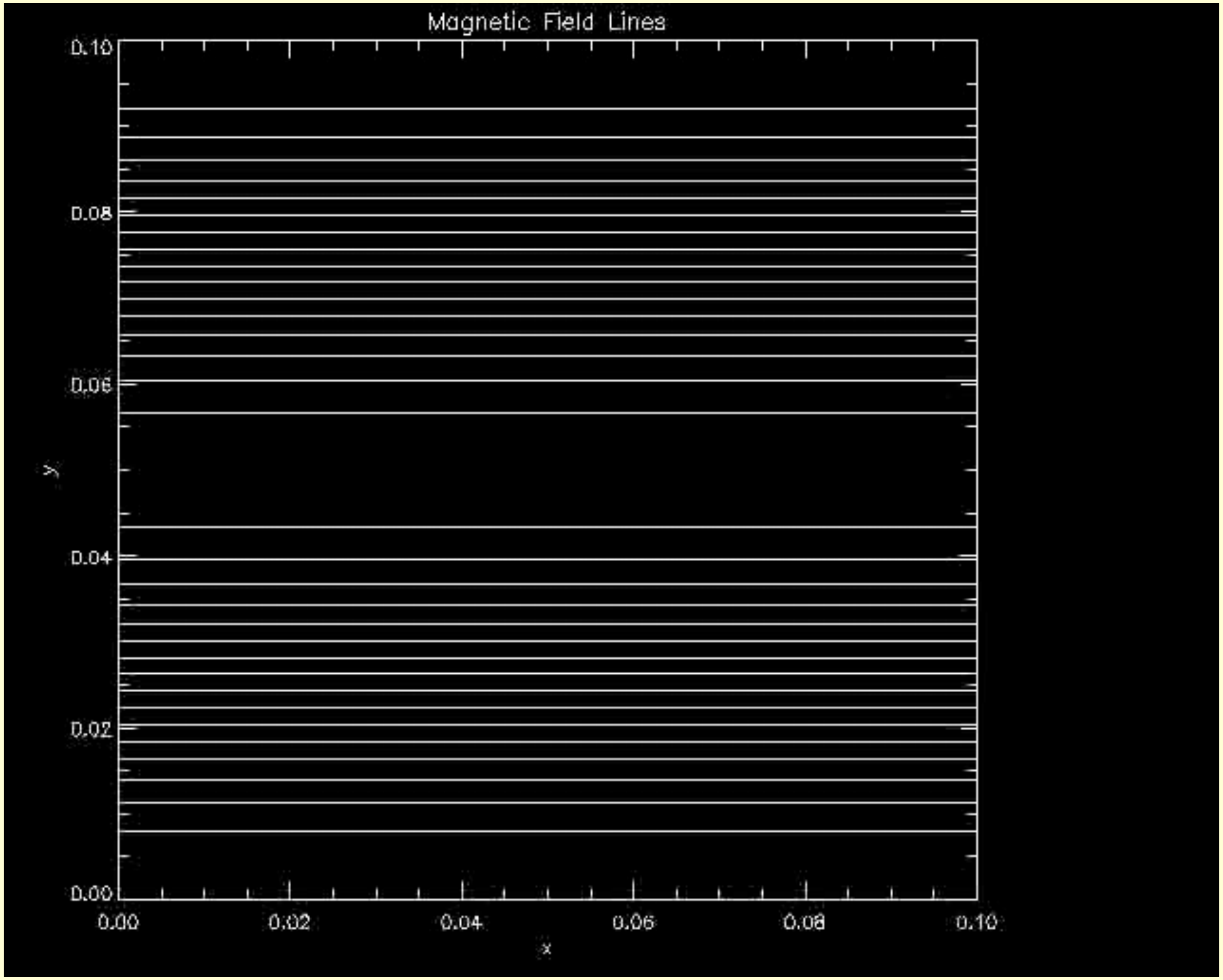
Effect of Finite B on Temperature Profile



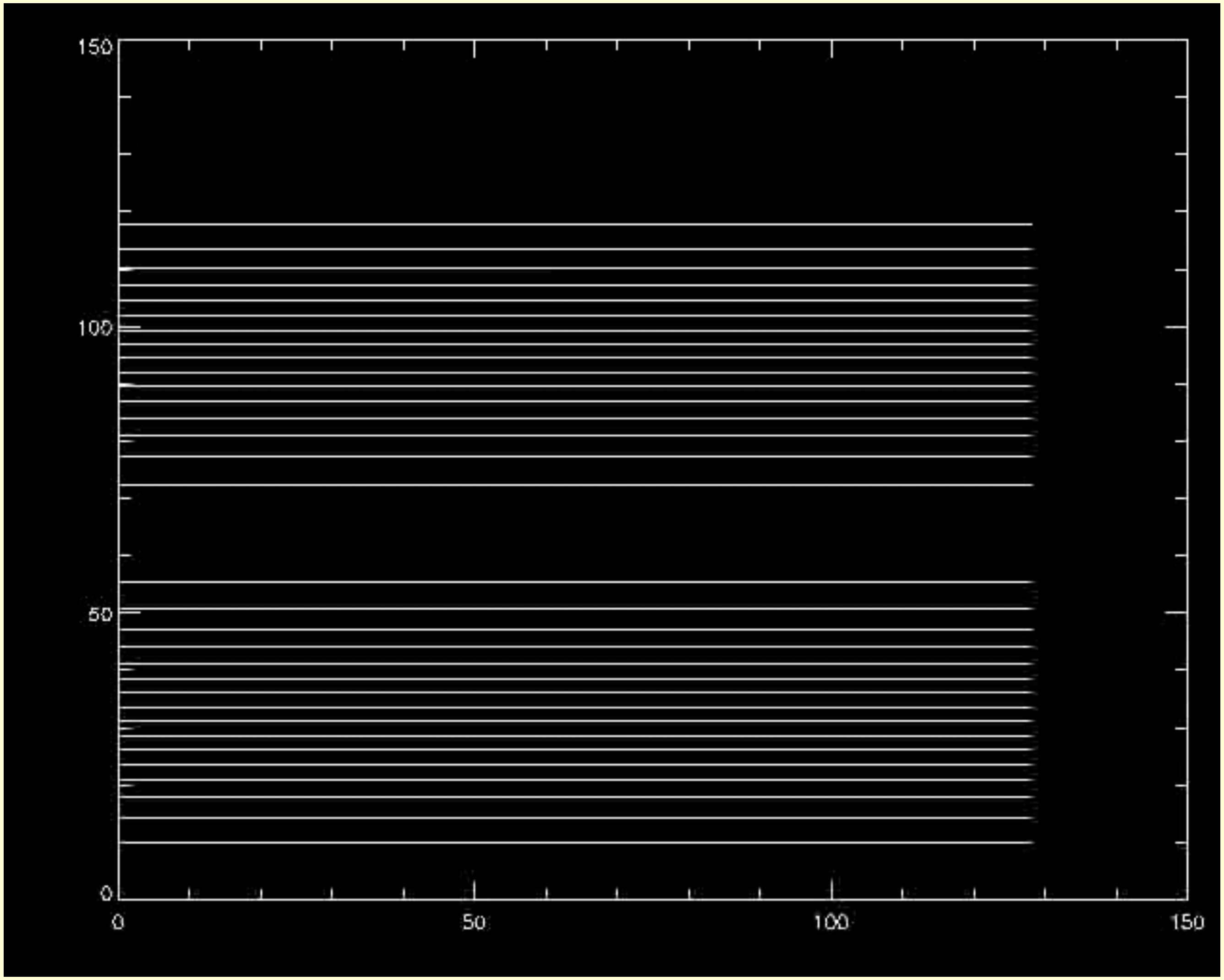
Stability Parameter:

$$\eta = \frac{k^2 V_A^2}{\sigma_{\max}^2}$$

Adiabatic Multimode Example



Conducting Boundaries

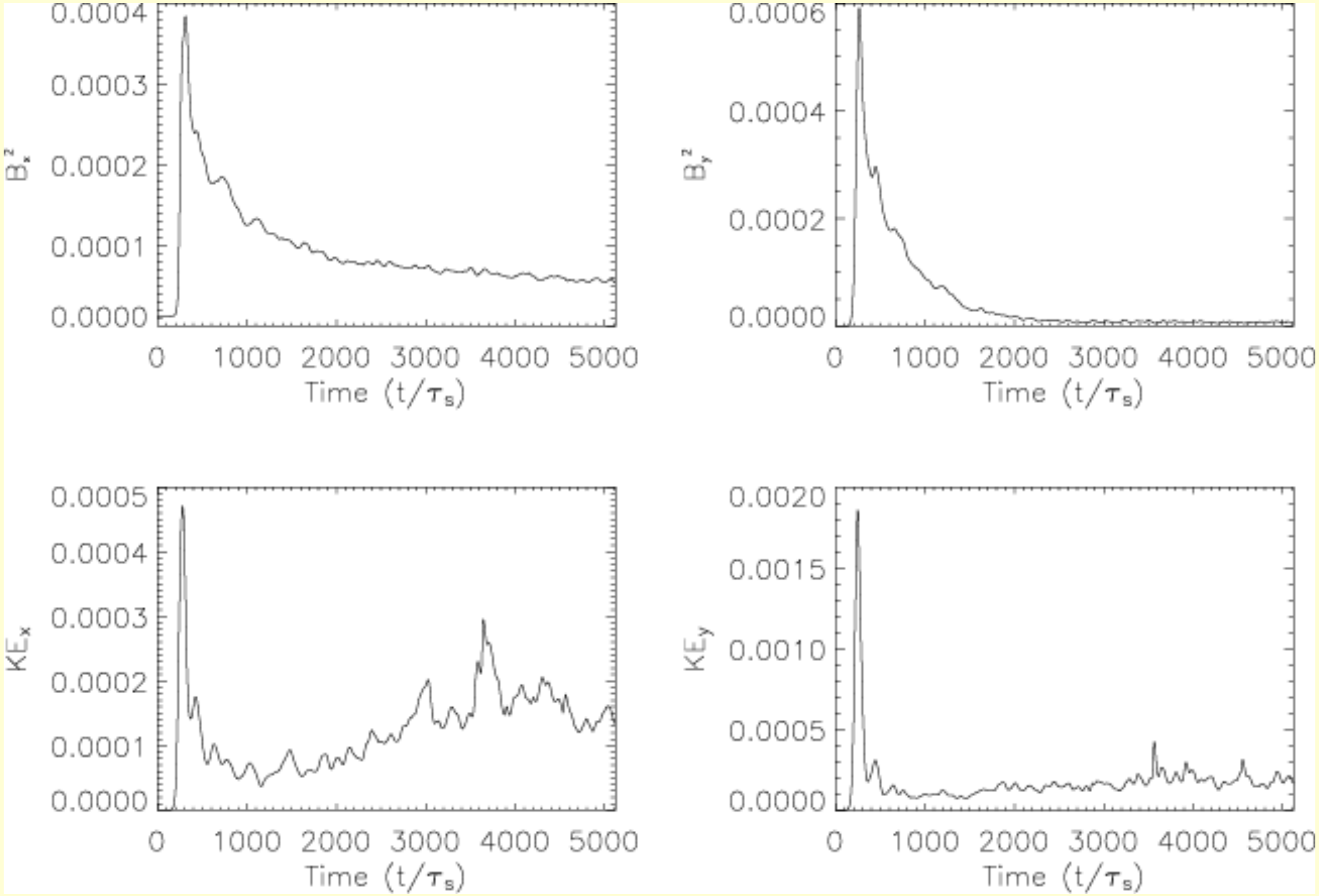


Magnetic Field Lines

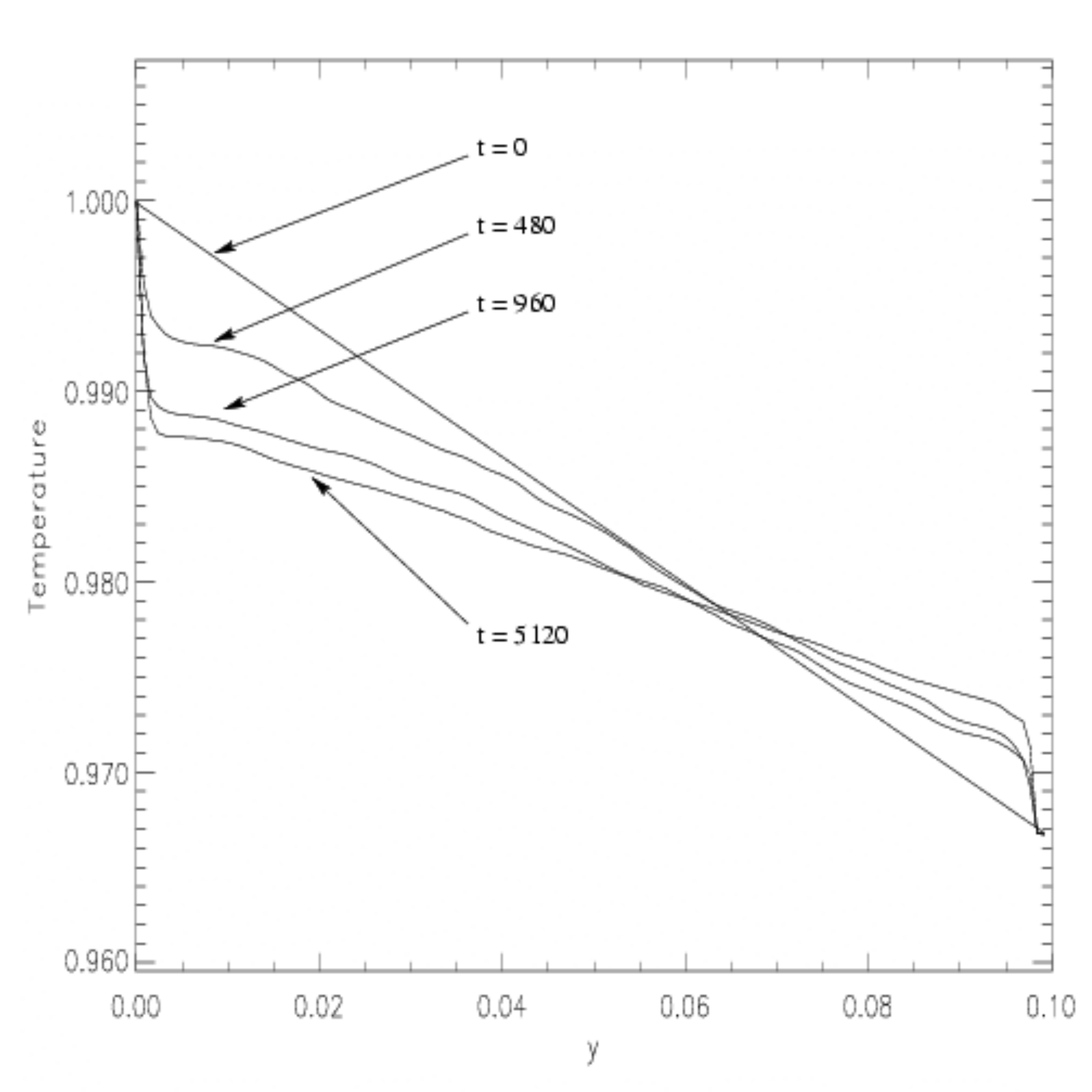
Conducting Boundary

Magnetic Energy Density

Kinetic Energy Density



Conducting Boundary Temperature Profiles



Extension to 3D

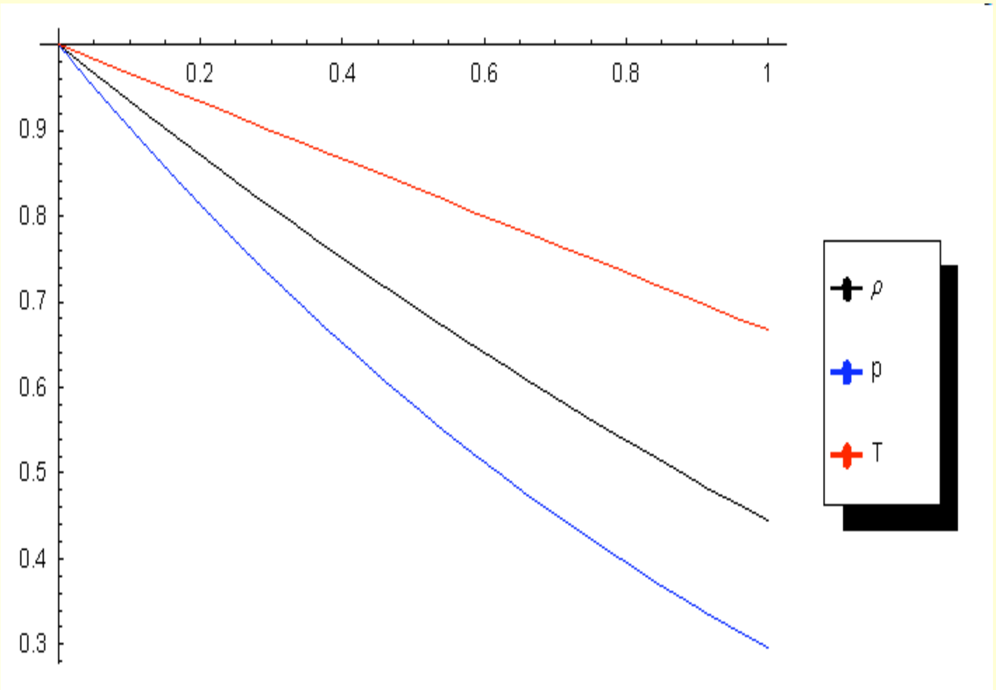
How to get there

- ATHENA is already parallelized for 3D
- Need to parallelize heat conduction algorithm
- Parallel scalability up to 2,048 processors

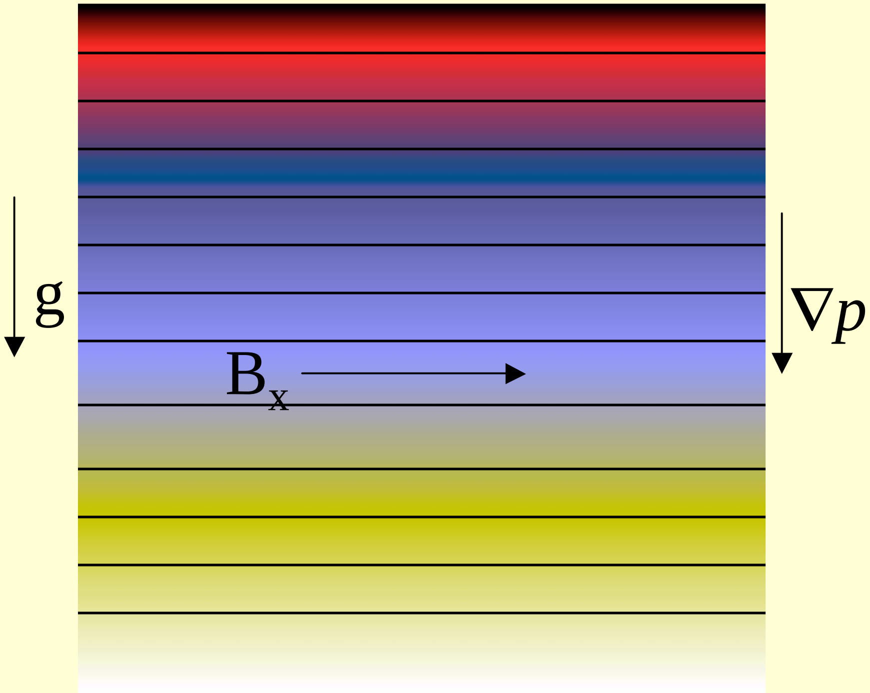
What can be studied

- Confirm linear and non-linear properties in 2D
- Convection is intrinsically 3D—measure heat conduction
- Possibility of a dynamo?

Initial Conditions



Pressure Profile



$$g(z) = -g_0$$

$$T(z) = (1 - T_0 z)$$

$$\rho(z) = (1 - T_0 z)^2$$

$$P(z) = (1 - T_0 z)^3$$

- Convectively Stable Atmosphere
- Ideal MHD (ATHENA)
- Anisotropic Heat Conduction (Braginskii)
- BC's: adiabatic or conducting at y-boundary, periodic in x