Occultation and Microlensing

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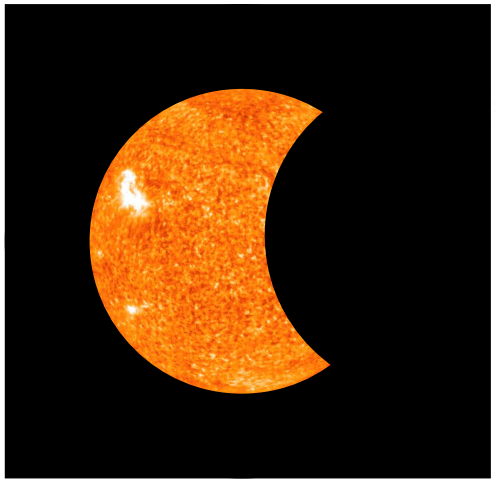
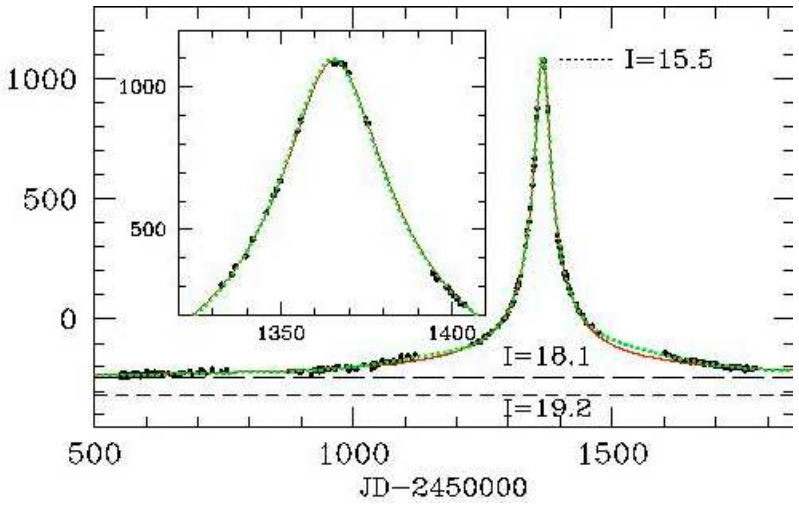
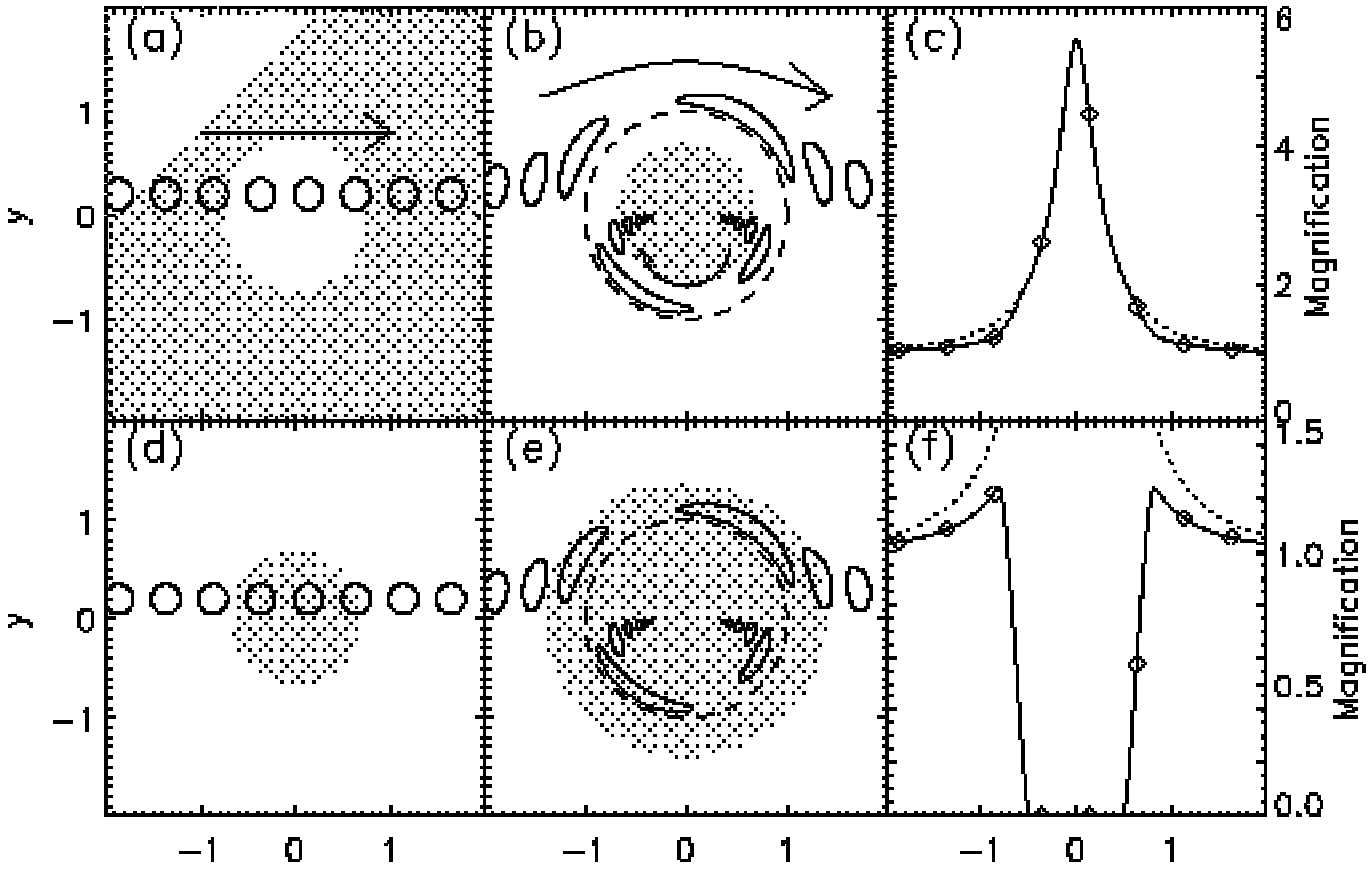




Lensing by an extended lens

$R_L < R_E$

$R_L > R_E$



$$R_E = \sqrt{\frac{4GM}{c^2}\frac{D_L D_{LS}}{D_S}}$$

Agol (2002)

1. EQUATIONS

Here are the equations:

$$\mu_-^a = \frac{\Theta(r_S-\zeta_0)}{r_S^2} - \frac{1}{4\pi r_S^2}\left[2(1+r_S^2)\phi_1 - 4s\,\mathrm{sgn}(\alpha_3)\psi_0 + 4r_L^2\phi_2 + \sqrt{(\alpha_2-\alpha_0)(\alpha_0-\alpha_1)}\left(\sqrt{1+4/\alpha_0}-1\right) - G(\phi_0)\right], \quad (1)$$

$$G(\phi) = \frac{1}{\sqrt{\alpha_3(4+\alpha_1)}}\left[\alpha_3(4+\alpha_1)E(\phi,k_1) - (\alpha_1\alpha_2+3\alpha_3)F(\phi,k_1) + 4\alpha_1(1+r_S^2)\Pi(\phi,\alpha,k_1)\right], \quad (2)$$

$$\begin{aligned}
\alpha_0 &= \beta_L^2\\
\alpha_1 &= (\zeta_0-r_S)^2\\
\alpha_2 &= (\zeta_0+r_S)^2\\
\alpha_3 &= \zeta_0-r_S^2\\
\phi_0 &= \cos^{-1}\sqrt{\frac{\alpha_1(\alpha_2-\alpha_0)}{\alpha_0(\alpha_2-\alpha_1)}}\\
\phi_1 &= \cos^{-1}\left[\frac{\alpha_1+\alpha_2-2\alpha_0}{\alpha_2-\alpha_1}\right]\\
\phi_2 &= \cos^{-1}\left[\frac{\alpha_3+\alpha_0}{2\zeta_0\alpha_0^{1/2}}\right]\\
\alpha &= 1-\frac{\alpha_1}{\alpha_2}\\
k_1^2 &= \frac{4(\alpha_2-\alpha_1)}{\alpha_3(4+\alpha_1)}.
\end{aligned} \quad (3)$$

$$\mu_-^{\zeta_0^{\rm im}} = \mu_-^{\zeta_0^{\rm im}} + \frac{1}{\pi r_S^2}\left[\left(1+r_S^2-r_L^2\right)\cos^{-1}\frac{\beta_L}{2r_S} + \frac{\nu_2}{4}(\beta_L-\nu_1) - (1+r_S^2)\tan^{-1}\frac{\nu_2}{\nu_1}\right] \quad (4)$$

$$\begin{aligned}
\nu_1 &= \sqrt{4+\beta_L^2}\\
\nu_2 &= \sqrt{4r_S^2-\beta_L^2},
\end{aligned} \quad (5)$$

$$\mu_\pm^{\zeta_0^{\rm im}} = \frac{1}{\pi r_S^2}\left[r_S+(1+r_S^2)\tan^{-1}r_S\right] \pm \frac{1}{2}. \quad (6)$$

TABLE 1

MAGNIFICATION OF INNER IMAGE

	r_L	r_S	ζ_0	$\mu_-(\zeta_0)$
I.	0	0	$(0,\infty)$	μ_+^a
	$(0,1)$	0	$(0,\beta_L)$	
II.	$(0,1)$	$(0,\infty)$	$[\beta_L+r_S,\infty)$	0
	$[1,\infty)$	$[0,\infty)$	$[0,\infty)$	
III.	0	$(0,\infty)$	$[0,r_S)$	μ_-^a
	0	$(0,\infty)$	(r_S,∞)	
	$(0,1)$	$(0,\beta_L)$	$(r_S,\beta_L-r_S]$	
	$(0,1)$	$(0,\beta_L)$	$[0,r_S)$	
IV.	0	$(0,\infty)$	r_S	$\mu_-^{\zeta_0^{\rm im}}$
	$(0,1)$	$\left(0,\frac{1}{2}\beta_L\right]$	r_S	
V.	$(0,1)$	$[\beta_L,\infty)$	(r_S,β_L+r_S)	μ_-^a
	$(0,1)$	$[\beta_L,\infty)$	$(r_S-\beta_L,r_S)$	
	$(0,1)$	$(0,\beta_L)$	$(\beta_L-r_S,\beta_L+r_S)$	
VI.	$(0,1)$	$\left(\frac{1}{2}\beta_L,\infty\right)$	r_S	$\mu_-^{\zeta_0^{\rm im}}$
VII.	$(0,1)$	$[\beta_L,\infty)$	$[0,r_S-\beta_L]$	$\frac{1-r_L^2}{r_S^2}$

$$\mu_+^a = \frac{1}{4\pi r_S^2}\left[2(1+r_S^2)\psi_2 - 4s\,\mathrm{sgn}(\alpha_3)\psi_1 - 4r_L^2\phi_2 + \sqrt{(\alpha_2-\alpha_0)(\alpha_0-\alpha_1)}\left(\sqrt{\alpha_0/(4+\alpha_0)}+1\right) + G(\psi_0)\right], \quad (7)$$

$$\begin{aligned}
\psi_0 &= \cos^{-1}\sqrt{\frac{(\alpha_0-\alpha_1)(4+\alpha_2)}{(4+\alpha_0)(\alpha_2-\alpha_1)}},\\
\psi_1 &= \frac{\pi}{2}-\phi_0,\\
\psi_2 &= \pi-\phi_1+2\cos^{-1}\sqrt{\frac{\alpha_0(4+\alpha_0)}{\alpha_0(4+\alpha_1+\alpha_2)-\alpha_1\alpha_2}},
\end{aligned} \quad (8)$$

$$\begin{aligned}
\mu_-^c &= \frac{G(\pi/2)}{4\pi r_S^2} - \frac{1}{2},\\
\mu_+^c &= \frac{G(\pi/2)}{4\pi r_S^2} + \frac{1}{2},
\end{aligned} \quad (9)$$

$$\mu = \frac{\sqrt{4+r_S^2}}{6\Omega r_S^2}\left[\alpha_1\left((2+r_S^2)E(\phi_3,k_2) - 2F(\phi_3,k_2)\right) + \alpha_2 r_S\right] + \frac{1}{6\Omega r_S^2}\left(\eta^2\alpha(4+\beta_L^2)^{\frac{1}{2}} + \alpha_3\right)\left(\alpha_1\sqrt{\alpha_3} + \alpha_2 - \frac{3\gamma_2}{2r_S}\alpha_3\right) + \frac{2\gamma_2}{r_S^2\Omega}\left[\frac{\alpha_3}{8}(2+r_S^2) + \sinh^{-1}\left(\frac{r_S}{2}\right) + \sinh^{-1}\left(\frac{s\beta_L}{2}\right)\right], \quad (10)$$

where

$$\begin{aligned}
\alpha_1 &= 2(\gamma_1+2\gamma_2),\\
\alpha_2 &= 3(1-\gamma_1-\gamma_2)r_S - \frac{3\gamma_2}{2r_S}(2+r_S^2),\\
\alpha_3 &= r_S^2-\beta_L^2,\\
\Omega &= 1-\gamma_1/3-\gamma_2/6,\\
\phi_3 &= \cos^{-1}\left(-s\frac{\beta_L}{r_S}\right),\\
s &= \mathrm{sgn}(1-r_L),\\
k_2^2 &= \frac{r_S^2}{4+r_S^2}.
\end{aligned} \quad (11)$$

if $\phi_3 > \pi/2$, then $E(\phi_3,k_2) = 2E(k_2) - E(\pi-\phi_3,k_2)$ and $F(\phi_3,k_2) = 2K(k_2) - F(\pi-\phi_3,k_2)$.

$$\mu = \frac{1}{\Omega r_S^2}\left[\frac{\alpha_1}{3k_2}\left((2+r_S^2)E(k_2) - 2K(k_2)\right) + \frac{\alpha_2\pi}{3k_2} + \frac{4\gamma_2}{r_S^2}\sinh^{-1}\left(\frac{r_S}{2}\right)\right]. \quad (12)$$

For $\gamma_2 = 0$, this expression agrees with equation (A6) in ?).

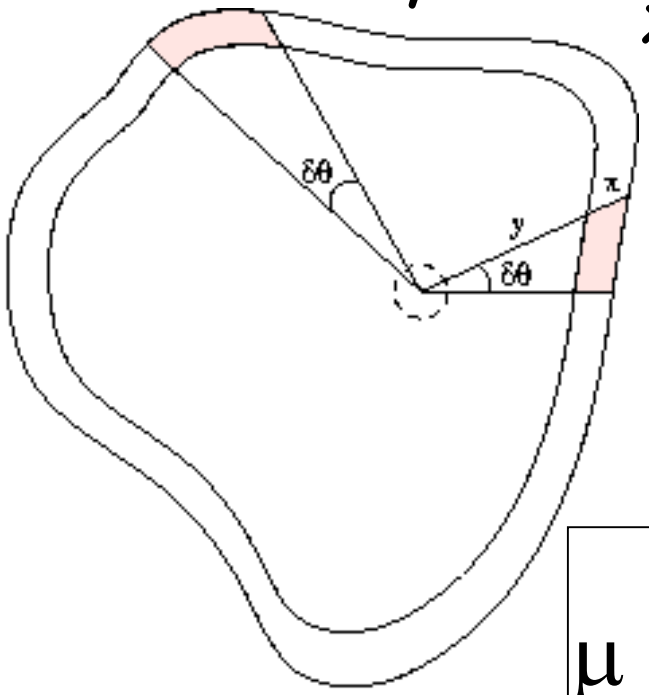
TABLE 2

MAGNIFICATION OF OUTER IMAGE

	r_L	r_S	ζ_0	$\mu_+(\zeta_0)$
I.	$(0,1)$	0	$(0,\infty)$	μ_+^c
	$[1,\infty)$	0	(β_L,∞)	
II.	$[1,\infty)$	$(0,\beta_L]$	$[0,\beta_L-r_S]$	0
III.	$[0,1)$	$(0,\infty)$	$[0,r_S)$	μ_+^a
	$[0,1)$	$(0,\infty)$	(r_S,∞)	
	$[1,\infty)$	$(0,\infty)$	$[\beta_L+r_S,\infty)$	
IV.	$[0,1)$	$(0,\infty)$	r_S	$\mu_+^{\zeta_0^{\rm im}}$
V.	$[1,\infty)$	$(0,\infty)$	$(\lvert r_S-\beta_L\rvert,\beta_L+r_S)$	μ_+^a
VI.	$[1,\infty)$	(β_L,∞)	$[0,r_S-\beta_L]$	$\mu_+^a+(1-r_L^2)/r_S^2$

Lensing of a large uniform source

$$y = x - \frac{1}{x} \Rightarrow y^2 = x^2 - 2 + \cancelto{0}{\frac{1}{x^2}}$$



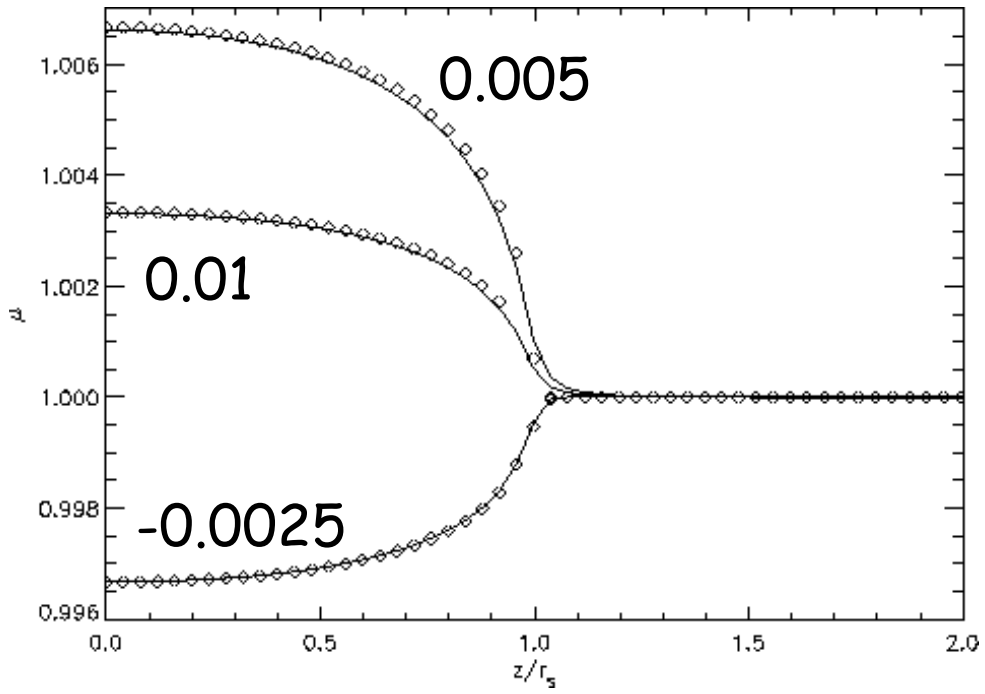
$$\mu = \frac{\int_0^{2\pi} \frac{1}{2} x^2 d\theta}{\int_0^{2\pi} \frac{1}{2} y^2 d\theta} = 1 + \frac{2\pi}{A} R_E^2$$

For a circular source:

$$\mu = 1 + \left(\frac{2R_E^2 - R_L^2}{R_S^2} \right) \Theta(1 - z)$$

Transit of a large limb-darkened source

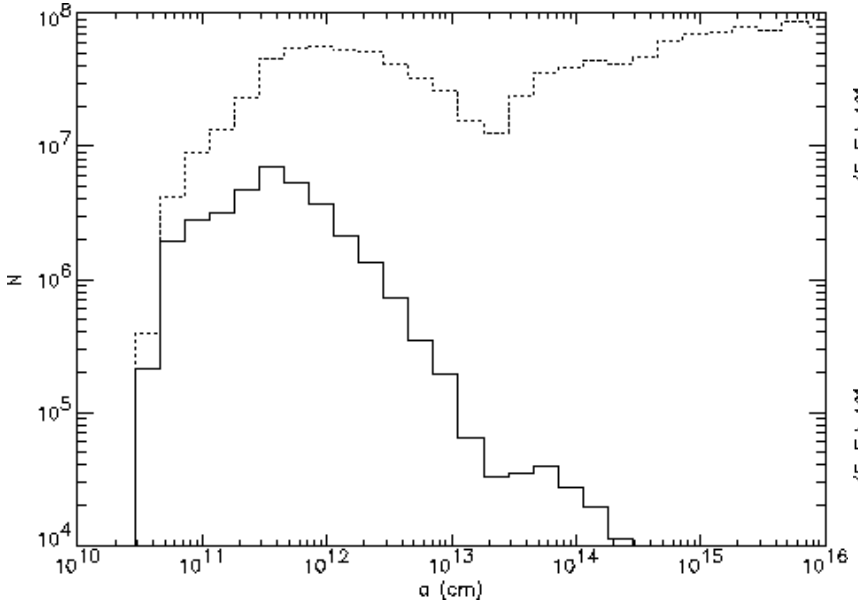
$$\mu = 1 + \frac{I(z)}{\langle I \rangle}\left(\frac{2R_E^2 - R_L^2}{R_S^2}\right)\Theta(1-z)$$



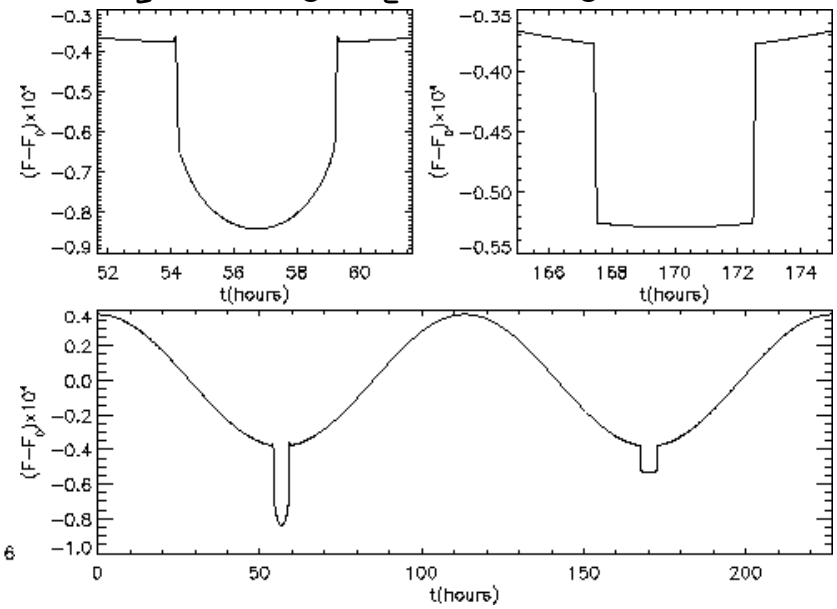
White Dwarf-Main Sequence Binaries

- Massive companion evolves off main-sequence - common envelope causes decrease in radius
- Peak at ~10^{12} cm - $R_E = 10^9 M^{1/2} a_{0.1}^{1/2}$

a-distribution for WD-MS binaries, uniform SFR



Lightcurve for $M_{WD}=0.5M_\odot$, $M_{MS}=1\ M_\odot$, $t_{WD}=2$Gyr, a=0.1AU, $R_L=0.01R_\odot$, $R_E=0.014R_\odot$



Kepler Mission - WD-MS binary detection possibilities

10°x10°, 3 yr mission, 11<V<14, 8σ detections

Solid lines: w/ stellar variability

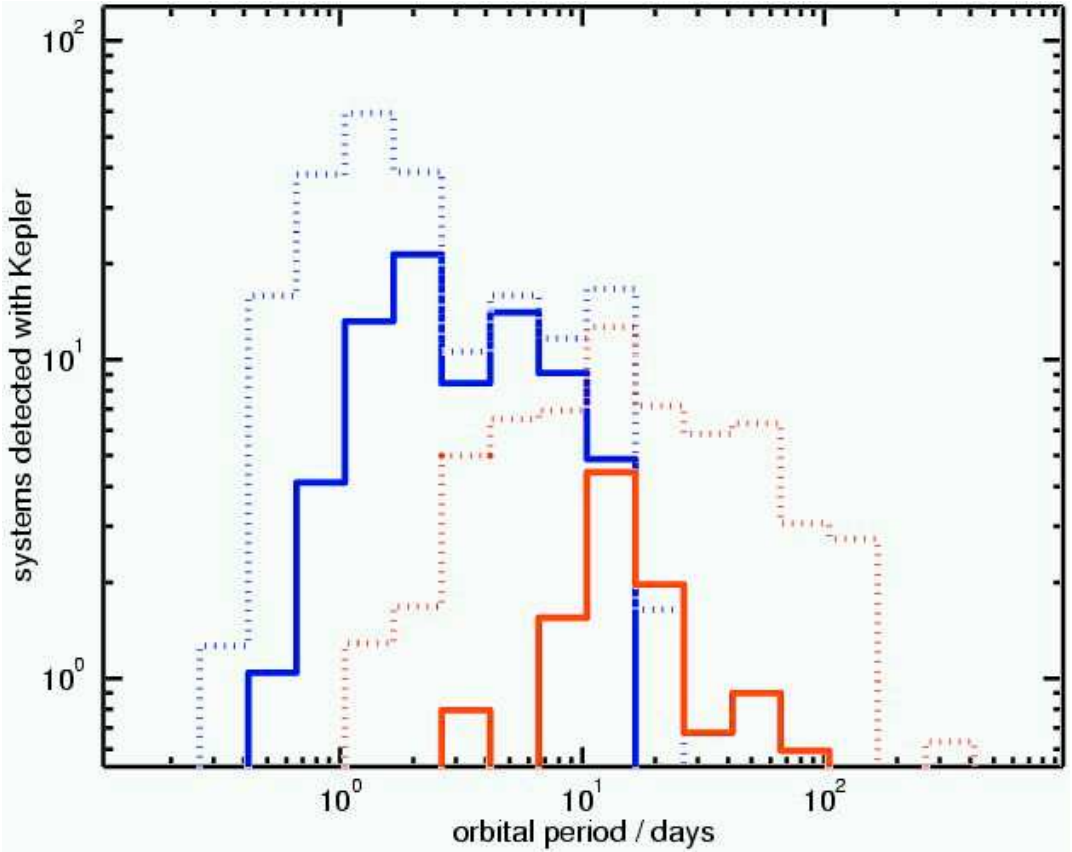
Dotted: w/o stellar variability

Blue: transit

(210, 76)

Red: microlensing

(61, 12)



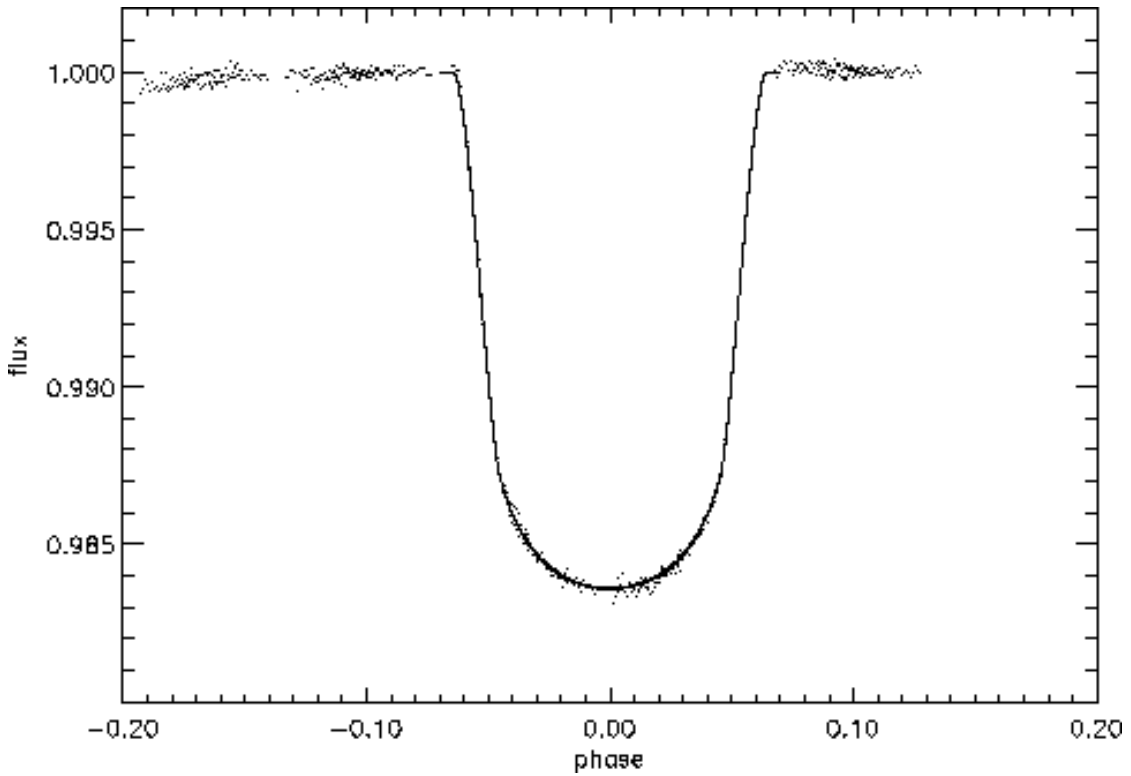
Farmer et al., in prep (see Beskin & Tuntsov for NS/BH)

Planetary Occultation Lightcurves

HD 209458, quadratic limb-darkening

$\frac{r_p}{r_*} = 0.1207 \pm 0.0003$ Mandel & Agol (2002)

HST Data from Brown et al. (2001), solid-line is best-fit model w/ quadratic limb-darkening



Conclusions

- Occultation & microlensing are different limits of the same phenomenon - inner image is occulted as lens & source separate
- Lenses in front of large sources add area of $2\pi R_E^2$ *independent of source size, shape, or lens position* (applies to WD, NS, or BH binaries)
- Lenses in front of large limb-darkened stars can be approximated by adding flux of $2\pi I(z) R_E^2$
- ~10^2 WD/MS can be found with searches for terrestrial planets - can be used to measure M/R relation originally predicted by Chandrasekhar
- Planetary transits can be used to accurately measure the planetary radius/star ratio