Incorporating Calibration Errors in Parameter Estimation Analysis

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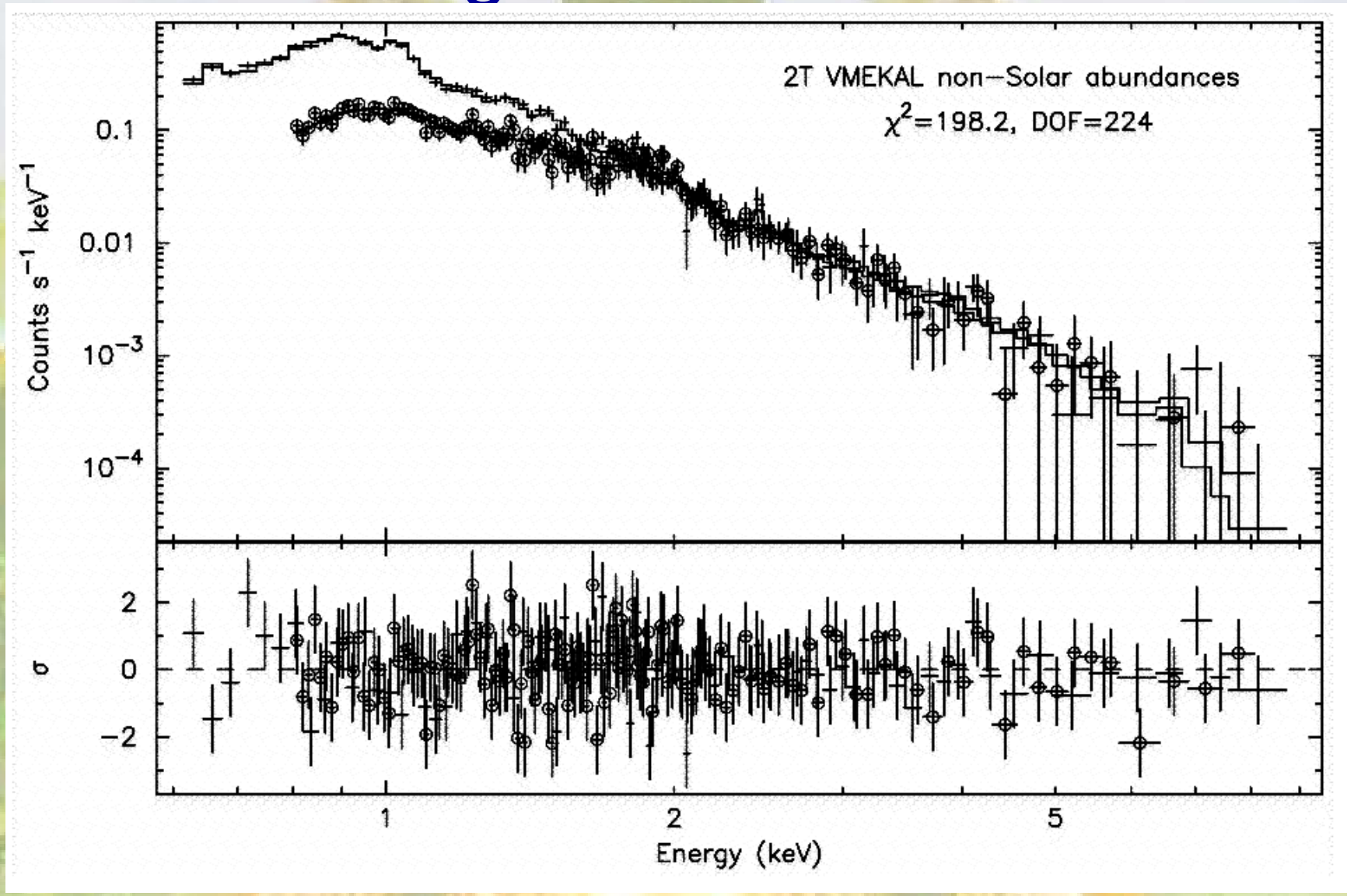
1. Correlated uncertainties
2. Study goals and methods: Monte Carlo
3. ACIS S3 example: model parameter estimation errors from effective area uncertainties
4. Conclusions
5. Future

Calibration uncertainties and why we ignore them



Correlated Uncertainties…

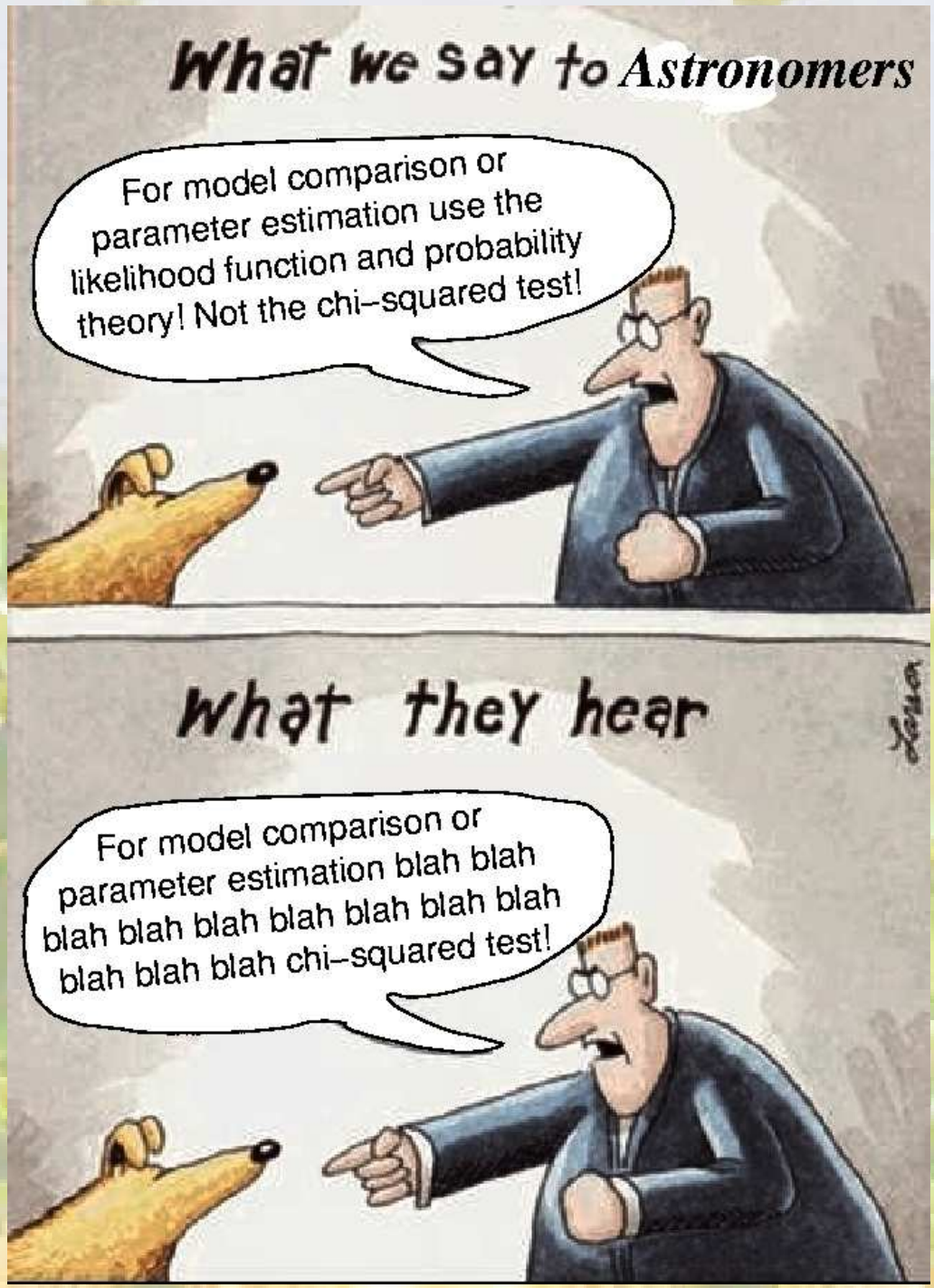
- Main problem is that uncertainties are **correlated** in very complicated ways
- There are no easy statistical formulae to apply

$$\begin{pmatrix} \sigma_{1,1} & \sigma_{2,1} & \cdots & \sigma_{n-1,1} & \sigma_{n,1} \\ \sigma_{1,2} & \sigma_{2,2} & & & \\ \vdots & & \ddots & & \vdots \\ \sigma_{1,n-1} & & & \sigma_{n-1,n-1} & \sigma_{n,n-1} \\ \sigma_{1,n} & & \cdots & \sigma_{n-1,n} & \sigma_{n,n} \end{pmatrix}$$

E.g. $\square_{i,j}$ represents relative uncertainty between channels i and j

" I haven't ever dealt with the problem of correlated uncertainties so I'm afraid I can't be much help. Good luck!"
– Keith Robinson (of Bevington & Robinson)

What we say to dogs
Okay, Ginger! I've had it! You stay out of the garbage! Understand, Ginger? Stay out of the garbage, or else!
What they hear
blah blah GINGER blah blah blah blah blah blah GINGER blah blah blah blah blah...


What we say to Astronomers
For model comparison or parameter estimation use the likelihood function and probability theory! Not the chi-squared test!
What they hear
For model comparison or parameter estimation blah blah blah blah blah blah blah blah blah blah chi-squared test!

Study Goals

- Use knowledge of instrument response uncertainties to **define limiting accuracy** of parameter estimation (eg T, N_H, [M/H], α)
- **Develop methods** and data for routine application of response uncertainties in parameter estimation analyses

Approach

- Analytical solutions difficult: use Moore's law and wait until Monte Carlo techniques are possible
- Simulate 100's-1000's+ of response functions that sample nominal response and its uncertainties
- Repeat parameter estimation for each response and examine distributions of "best fit" parameters

Sampling Response Errors: ACIS S3

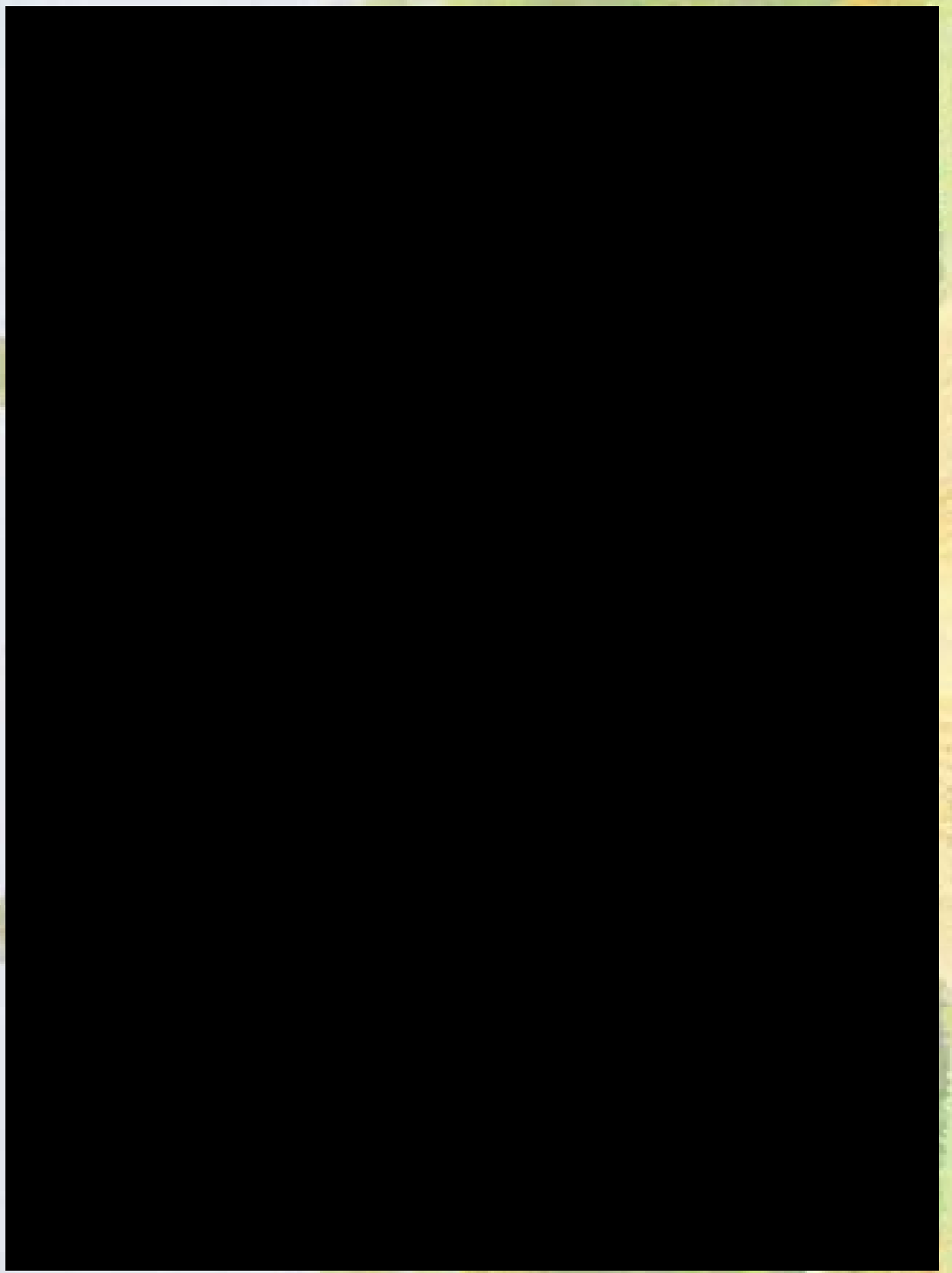
Uncertainties in Photon Path

HRMA: geometry, obscuration, reflectivity, scattering

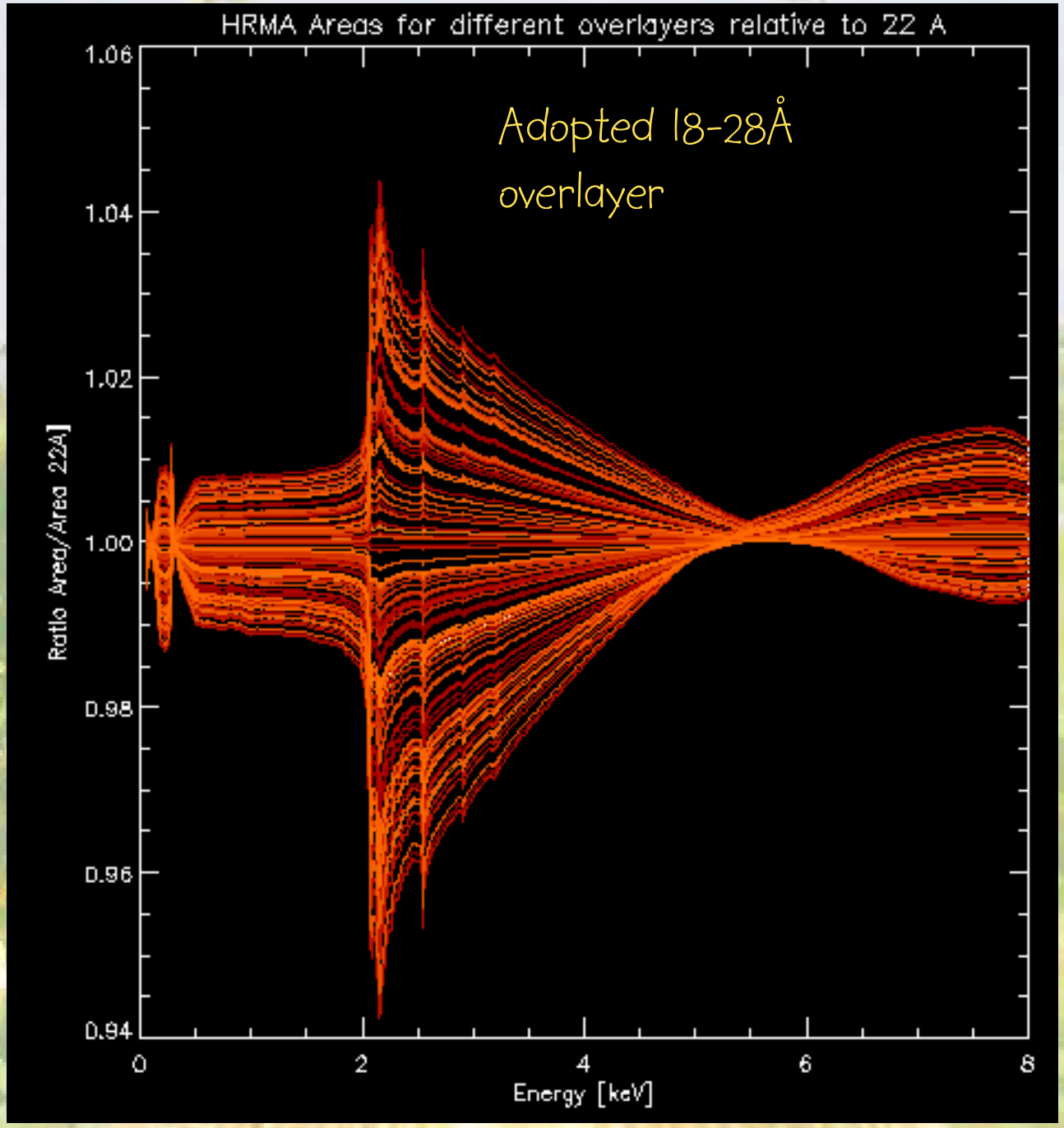
ACIS OBF: transmittance, contamination

ACIS: QE (CTI, dead time, cosmic rays, electronics…)

- Use parameterised instrument models where available and vary parameters
  - ACIS QE model
  - HRMA for different C overlayers (18-28Å)
- Derive a "fudge vector" - a perturbation vs E by which to change effective area
- No RMF variations, yet

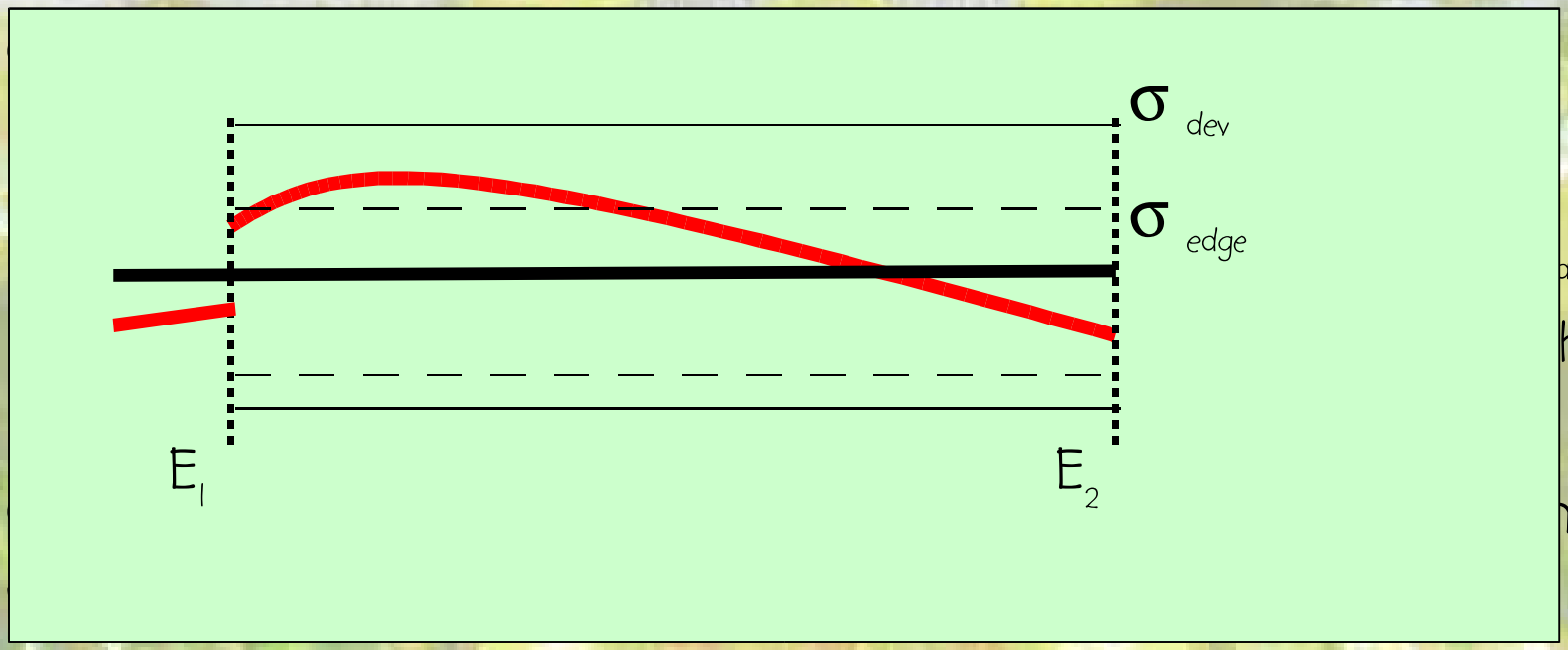


Adopted +/- 13% for ~10%
maximum deviation


HRMA Areas for different overlayers relative to 22 A
Adopted 18-28Å
overlayer
Ratio Area/Area 22A]
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0
2
4
6
8
Energy [keV]

Fudge Vector

- Function of E used to multiply nominal effective area curve to obtain different uncertainty-modified area each iteration
-
-



dev)

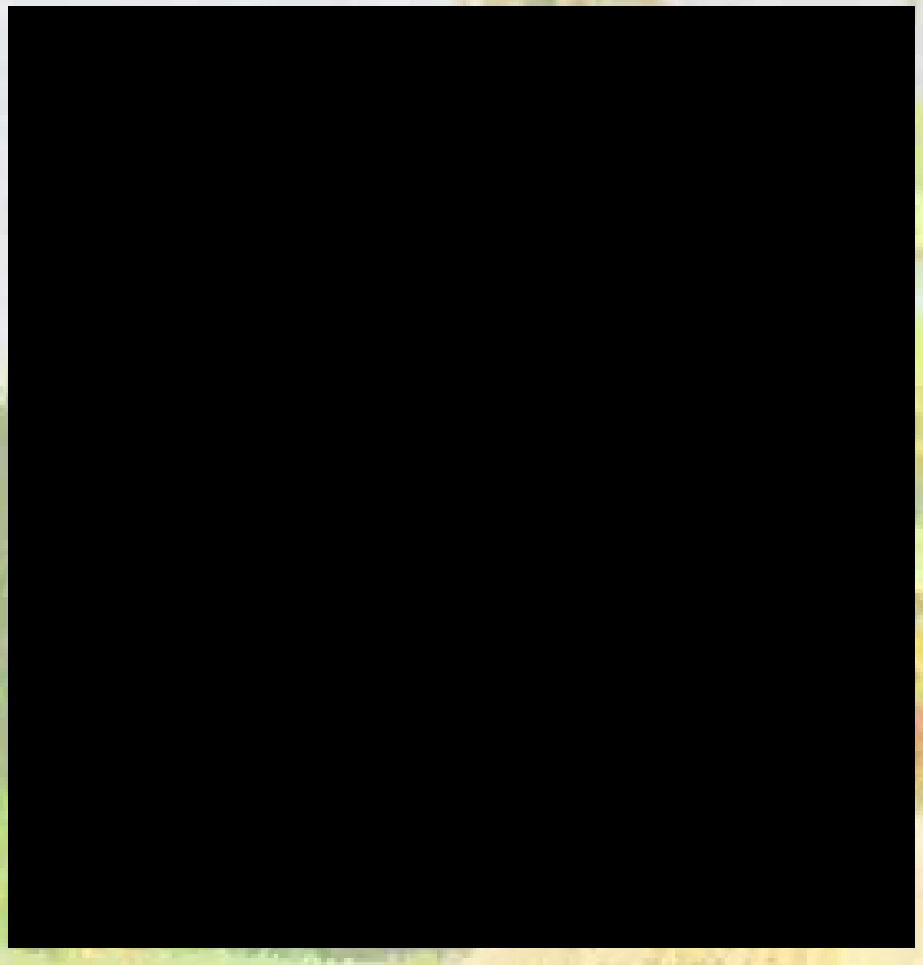
h range

model

Uncertainties Used

- Highly objective and meticulous assessment of σ_{edge}, σ_{dev} for each subassembly…
- Edges: 3-5%
- HRMA: 5->10% 0.5->10 keV
- Contamination: 15->3% 0.25->0.5 keV
- OBF: 7->3% 0.5->10 keV
- ACIS QE: 10-15% < 0.6 keV; 5% 0.6-1.8 keV; ~5%> 1.8 keV

How are the calibration uncertainties distributed?!!



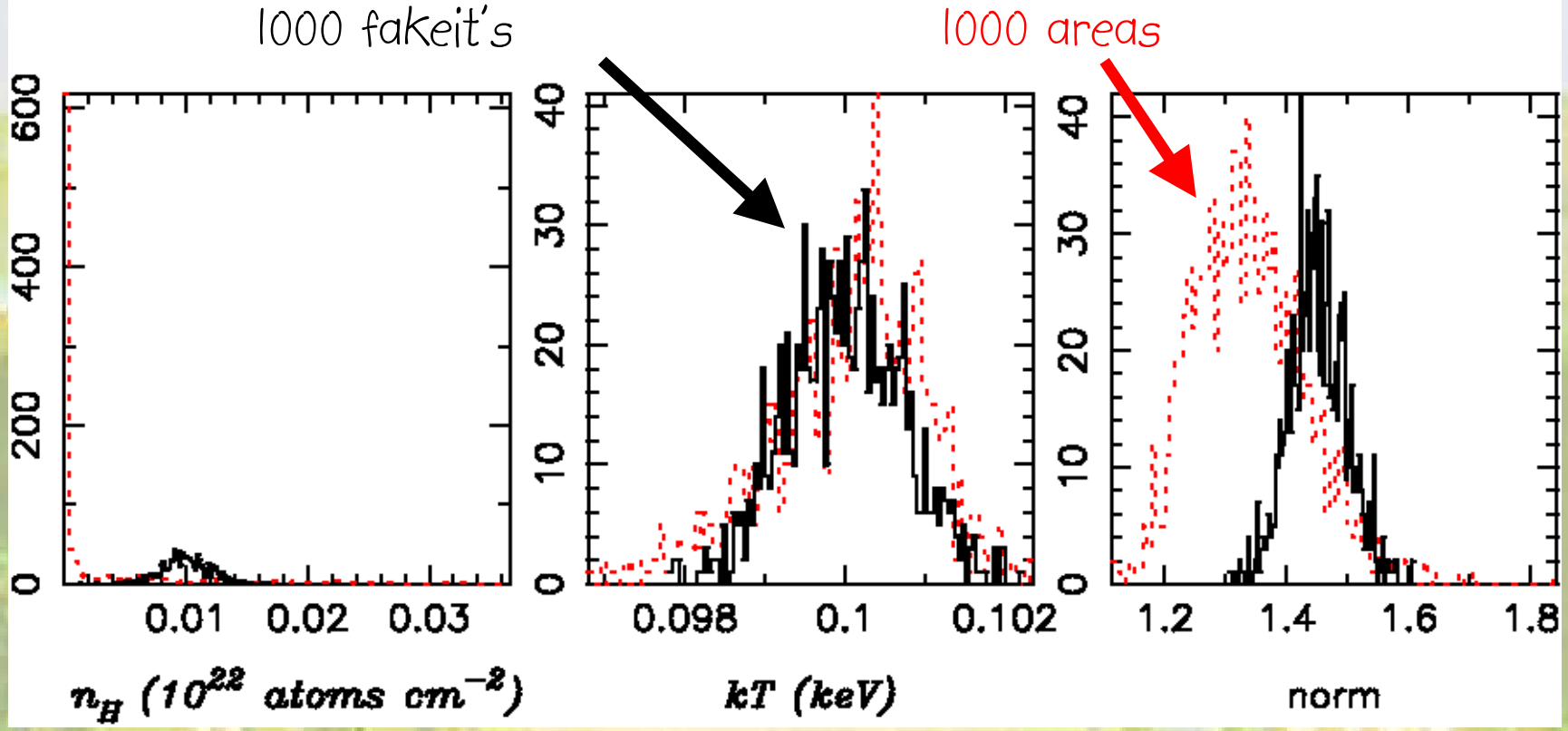
- Rigorous treatment requires knowledge of how uncertainties are distributed
- **Unknown!**
- Assume a truncated normal distribution -1σ to $+1\sigma$
  - Peaked at preferred value
  - Includes gut feeling!

QuickTime[a] and a
GIF decompressor
are needed to see this picture.

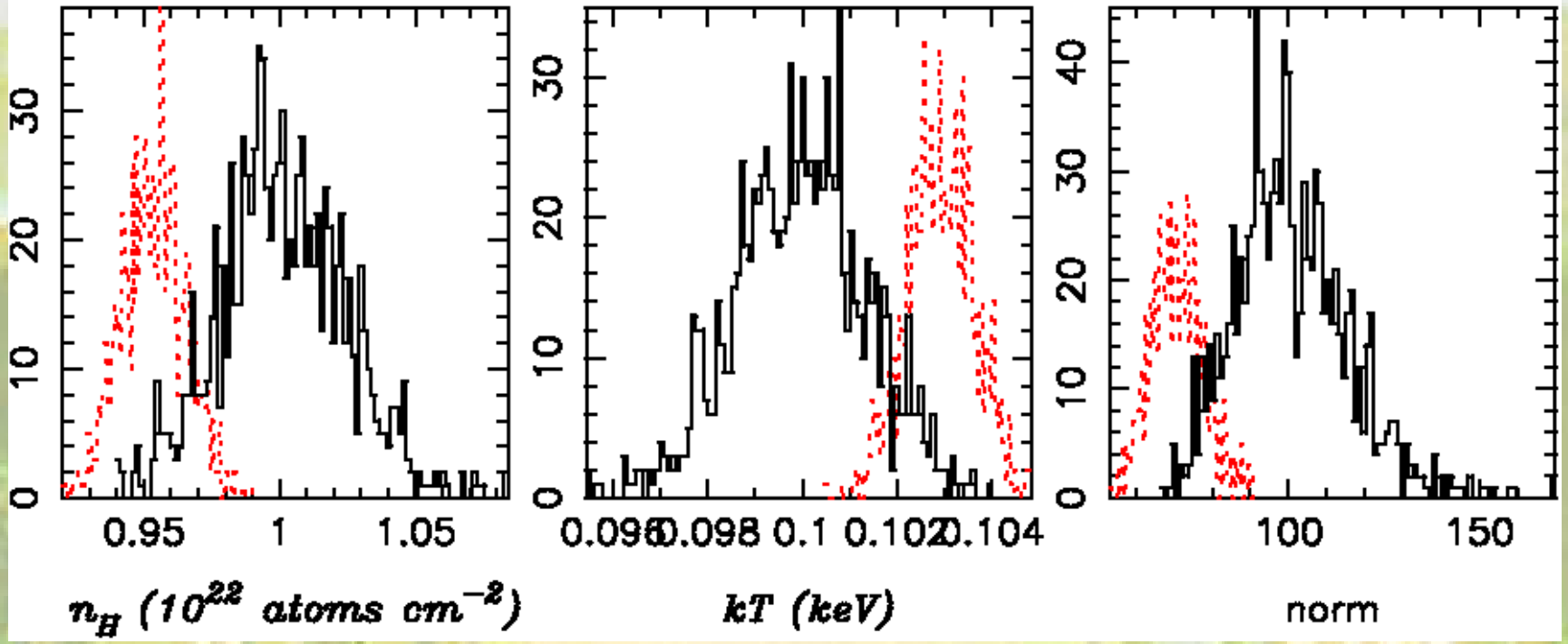
Parameter Estimation

- Simulate spectrum, ~10,000 counts
- Fit with different effective area realisation 1000 times
- XSPEC V12.2, Cash statistic
- Models: blackbody, MEKAL, powerlaw; all with ISM absorption
- Compare 1000 perturbed area fits with 1000 "fakeit" nominal area fits to probe area uncertainty effects with counting statistics

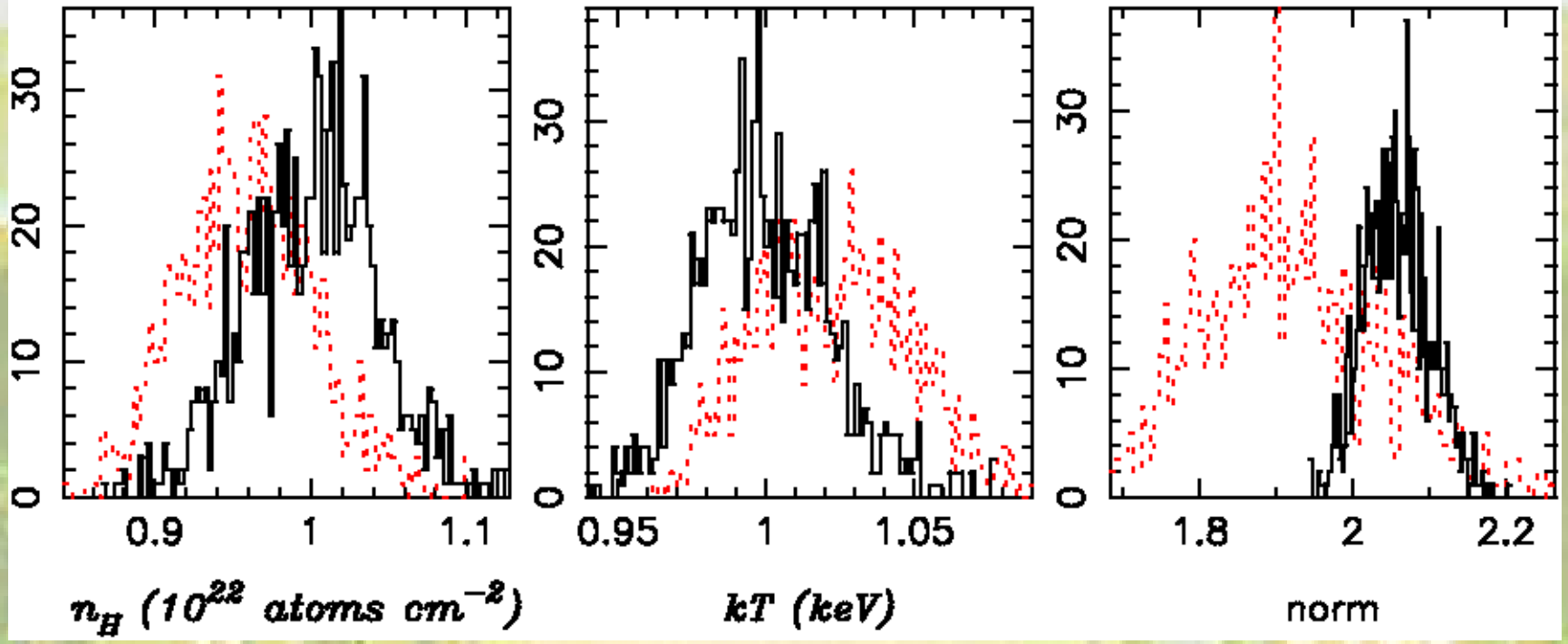
Absorbed Blackbody : $kT=0.1$, $n_H=10^{20}$



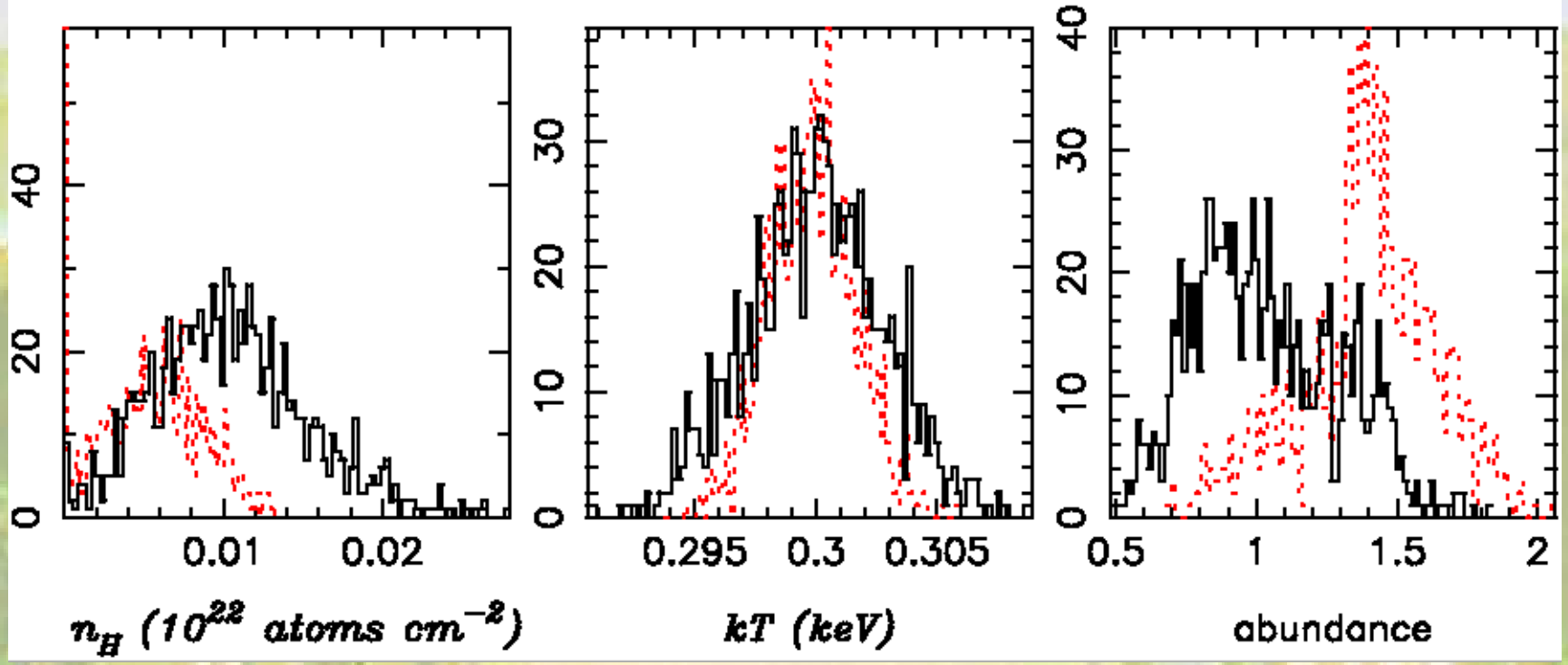
Absorbed Blackbody : $kT=0.1$, $n_H=10^{22}$



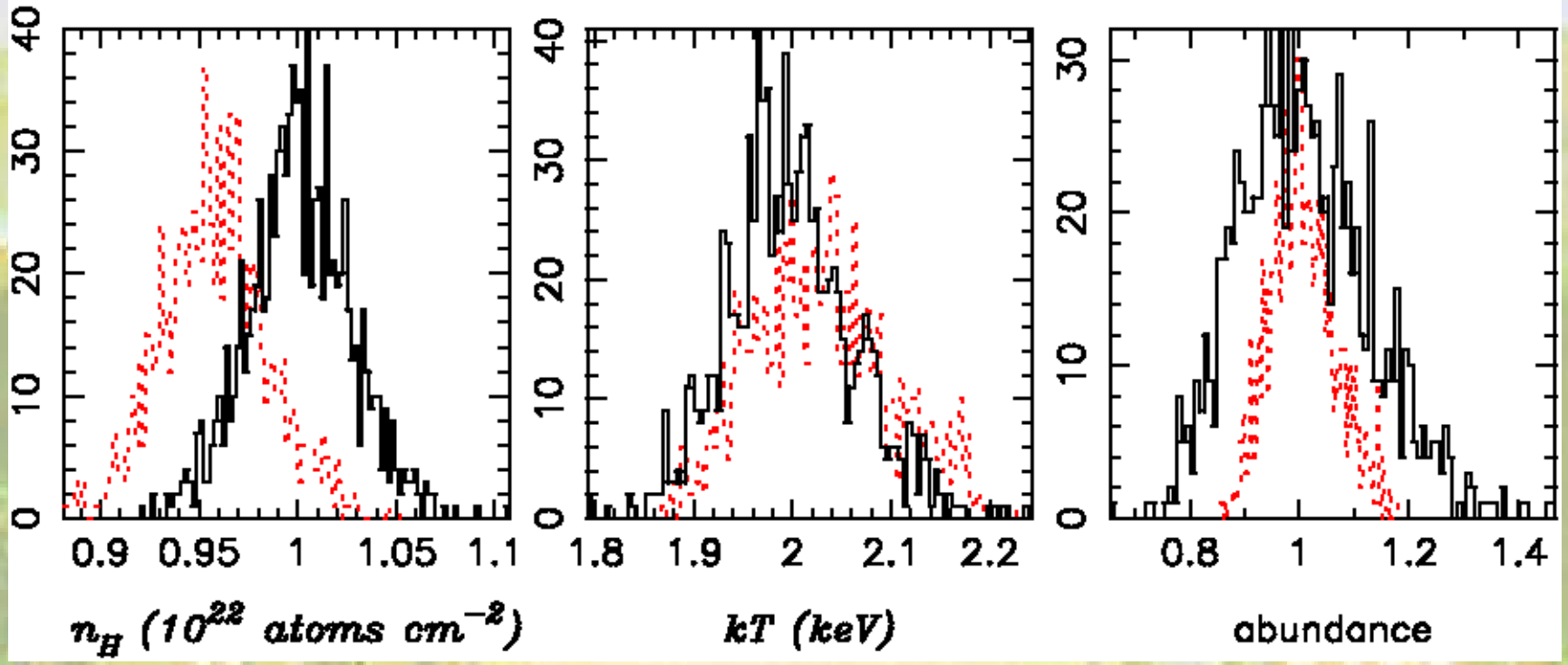
Absorbed Blackbody : $kT=1,\ n_H=10^{22}$



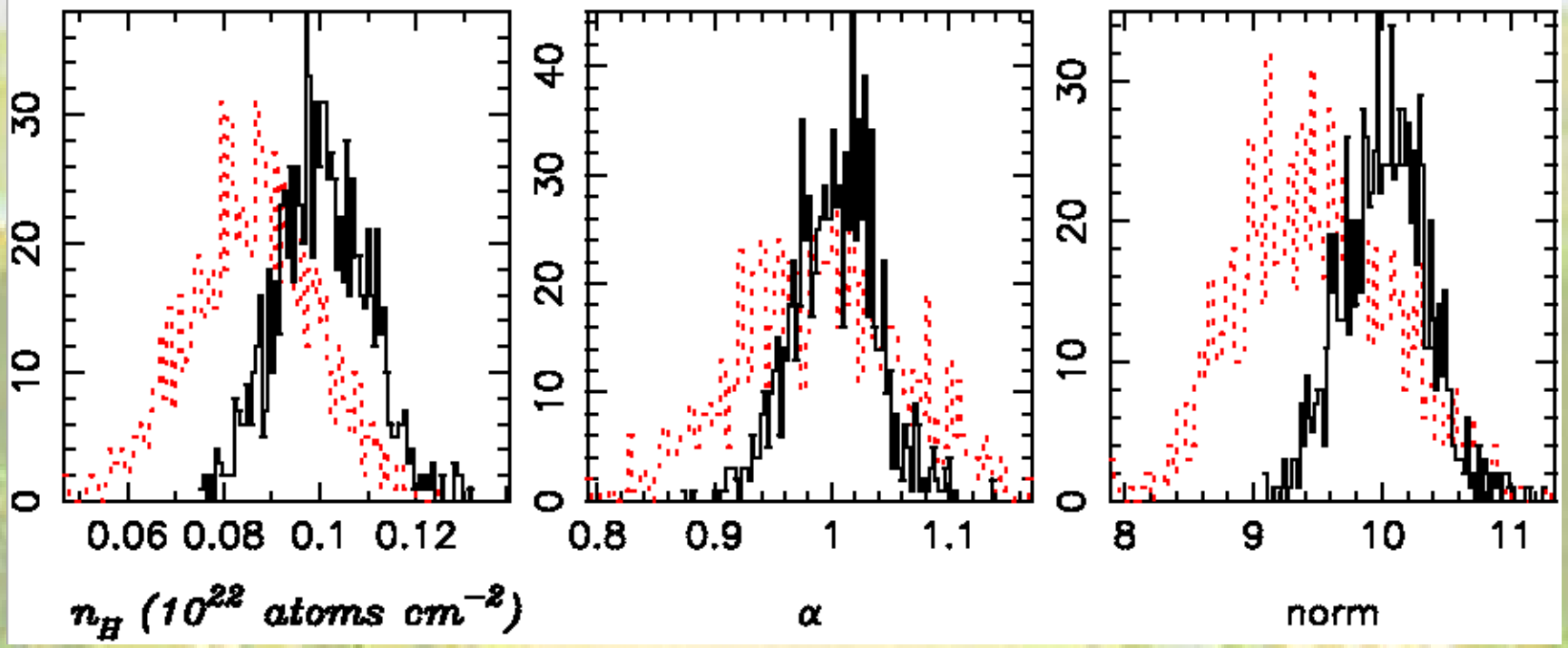
Absorbed Plasma : $kT=0.3,\ n_H=10^{20}$



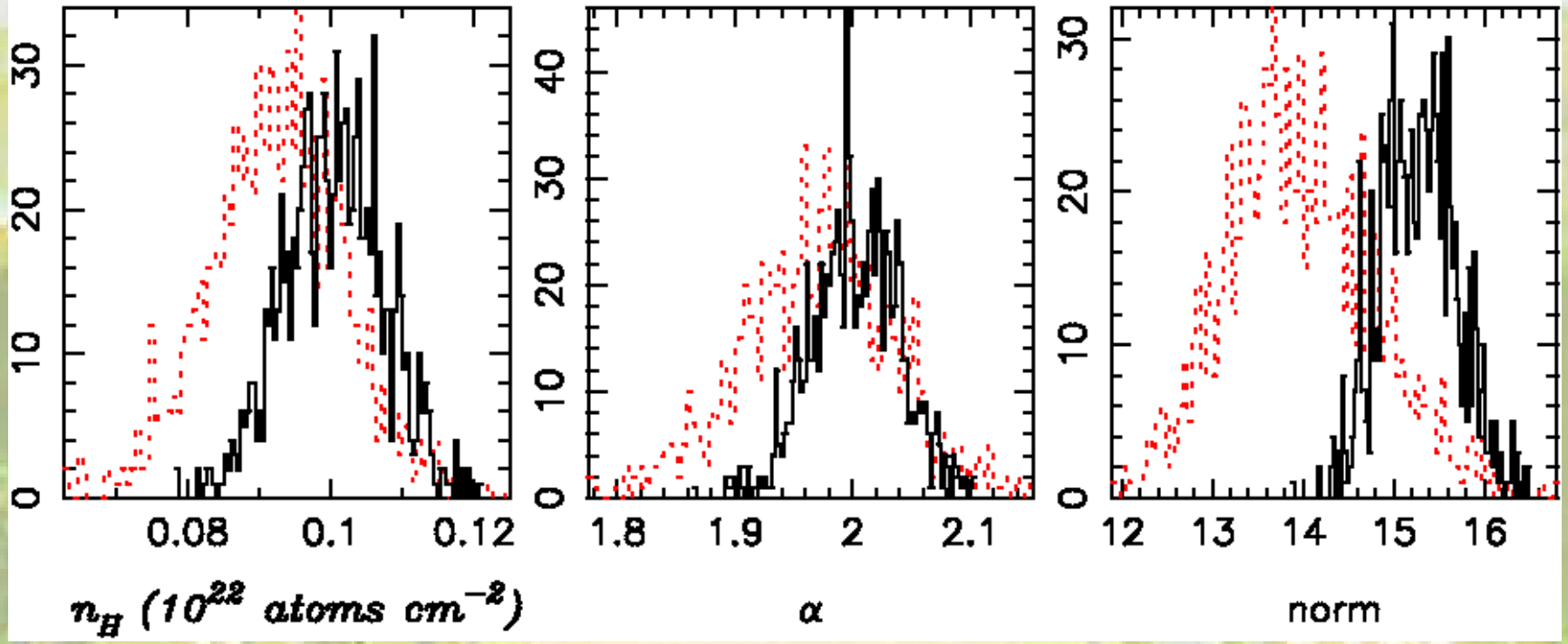
Absorbed Plasma : $kT=2,\ n_H=10^{22}$



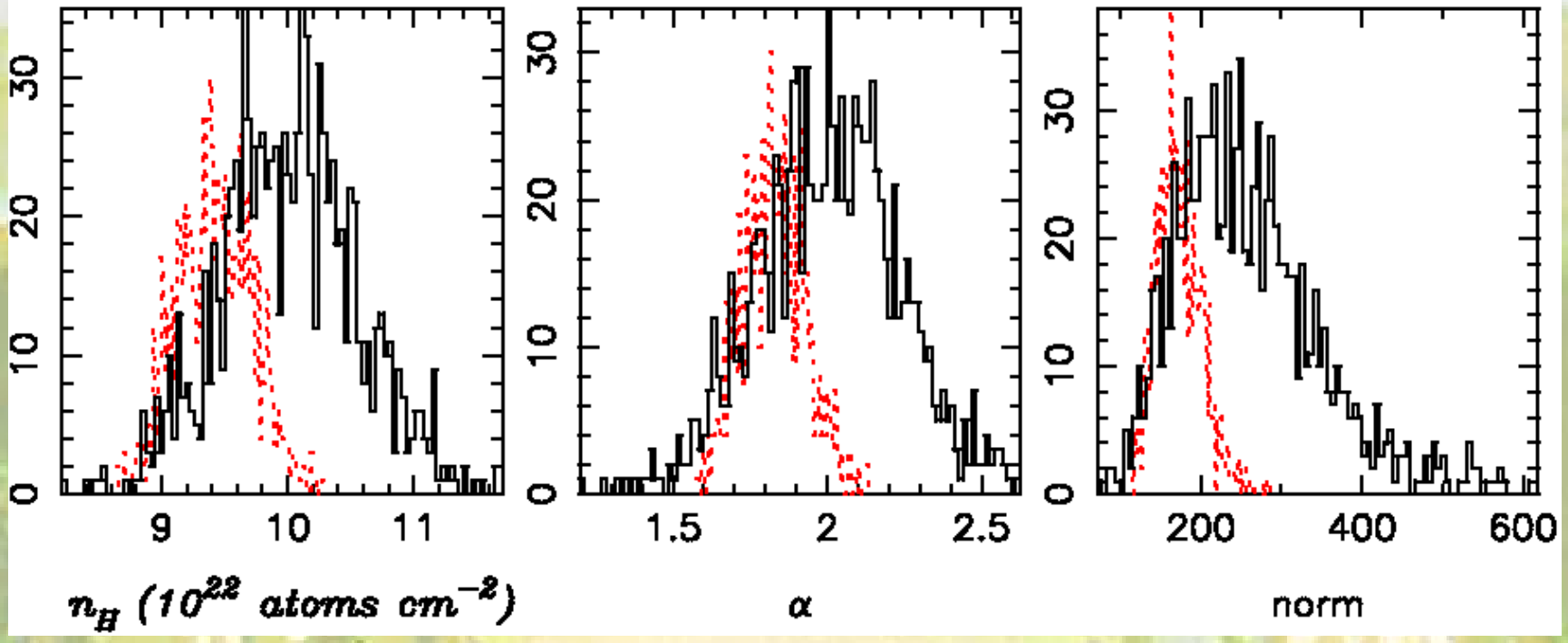
Absorbed Power Law : $\alpha=1$, $n_H=10^{21}$



Absorbed Power Law : $\alpha=2,\ n_H=10^{21}$



Absorbed Power Law : $\alpha=2$, $n_H=10^{23}$



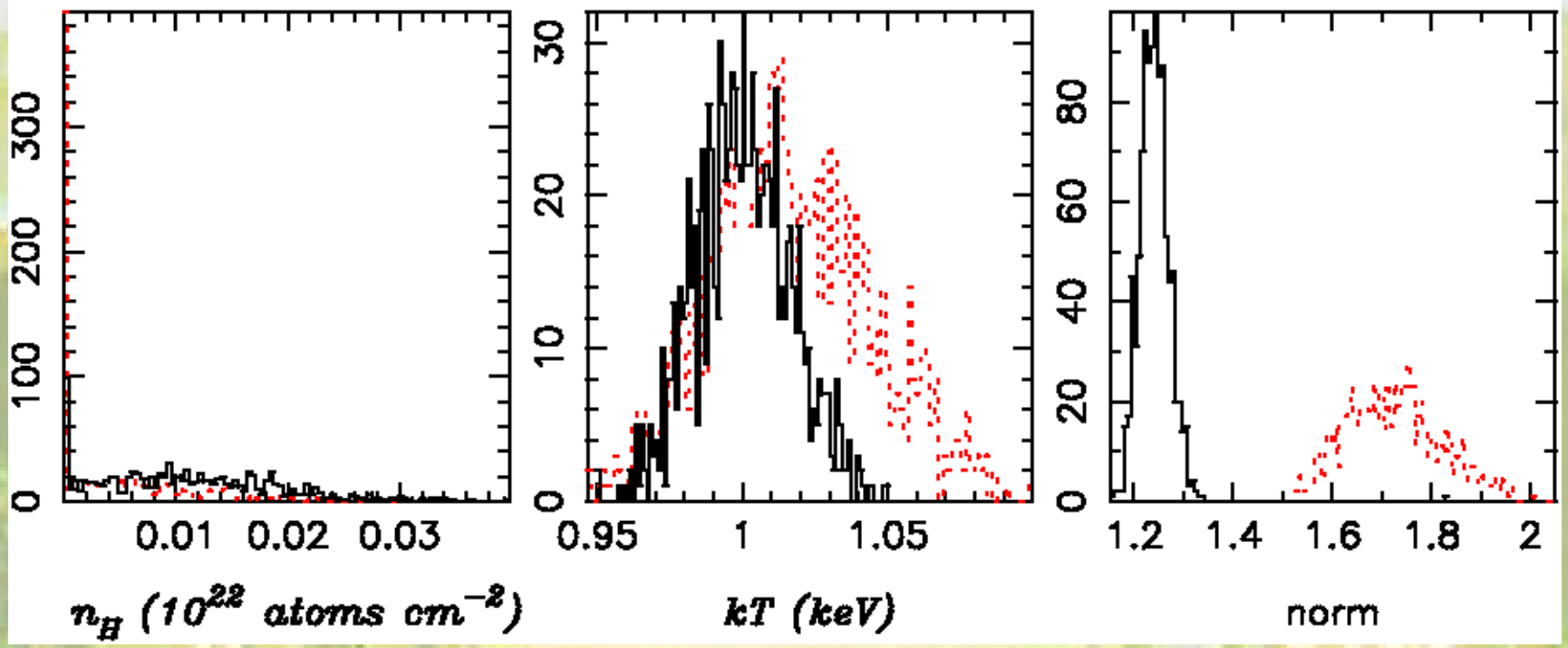
Conclusions

- We have developed Monte Carlo methods that can provide realistic means for
  - including very messy correlated calibration (**and model**) uncertainties in X-ray analysis.
  - understanding how well we need to calibrate different parts of instrument response
- Meticulous and objectively assessed ACIS uncertainties...==> mostly rather small parameter estimation errors, though some surprises and pathologies

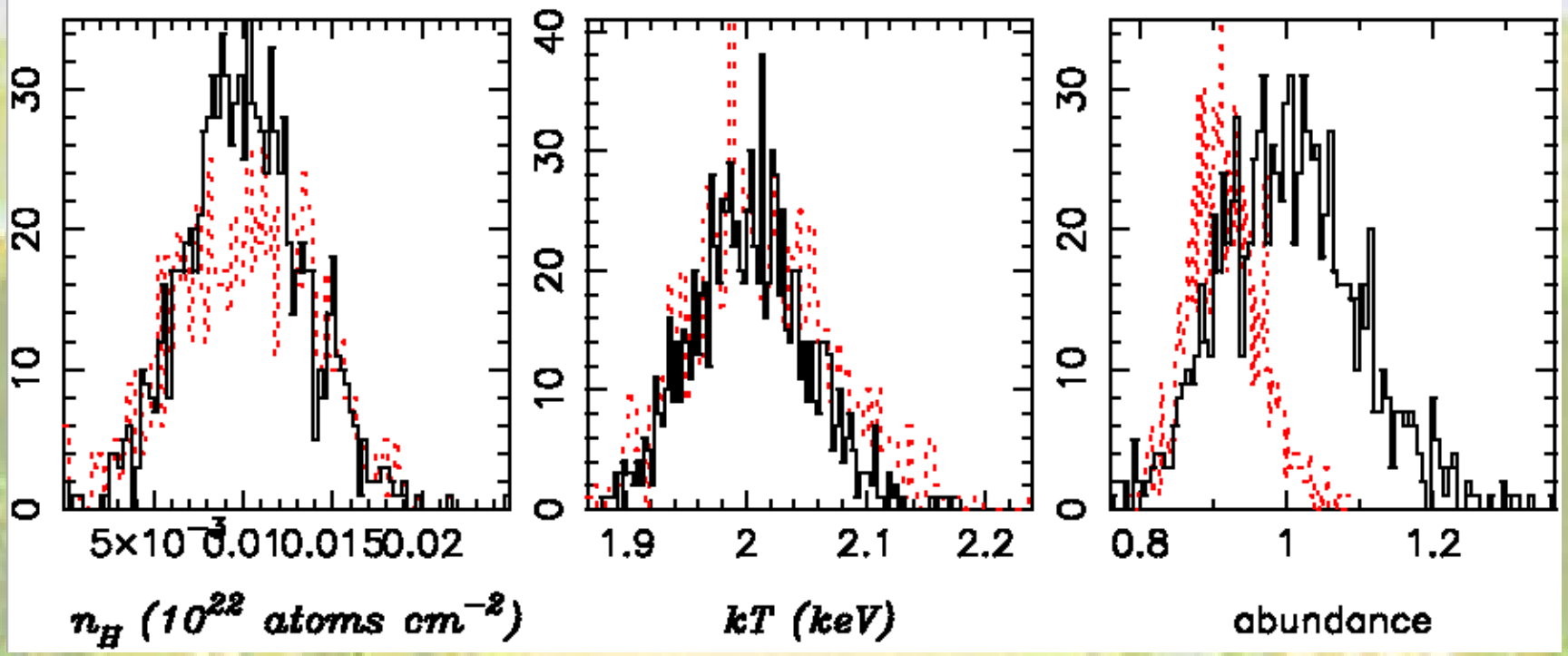
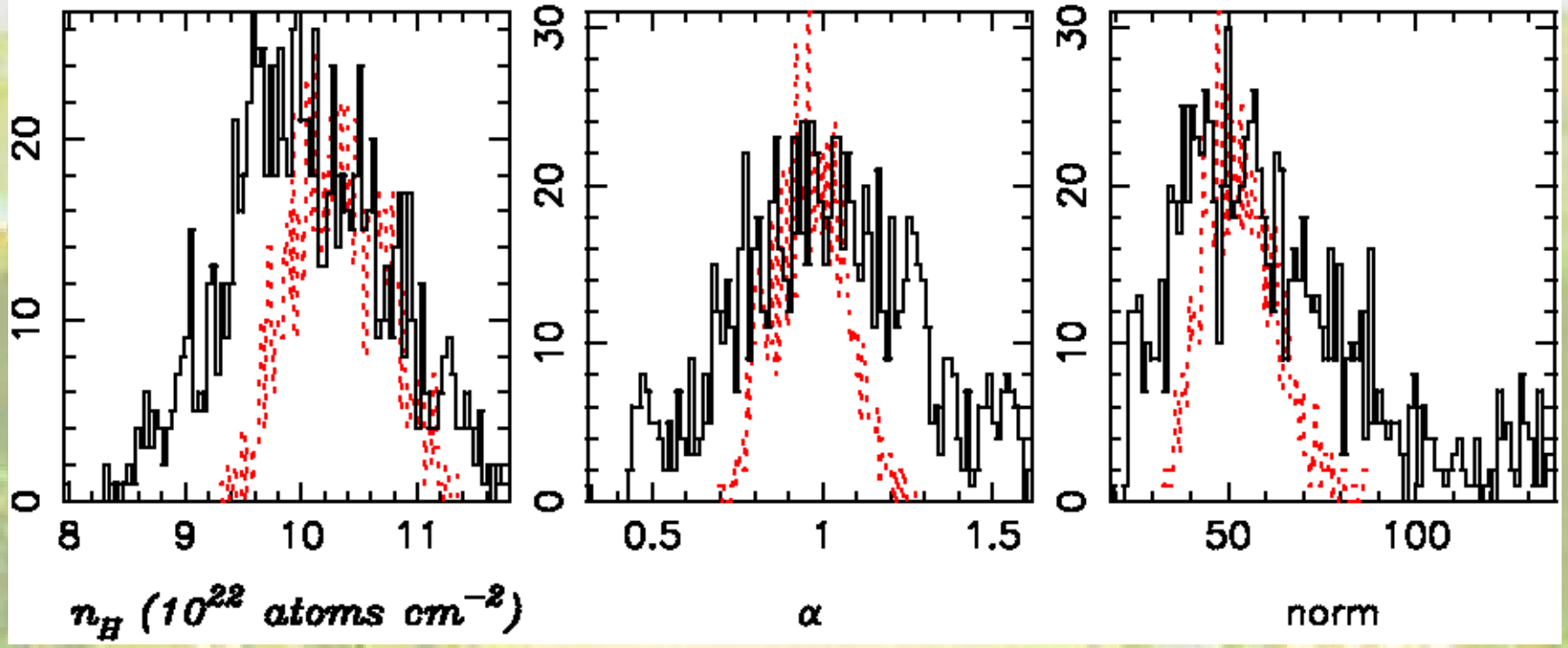
Future

- Monte Carlo approach adopted here presently cumbersome for general distribution
- Effective area uncertainties can be encoded in compressed form, eg PCA (Rima talk next)
- Sherpa development to use eg PCA-compressed uncertainty information to include in Markov Chain Monte Carlo parameter estimation

Absorbed Blackbody : $kT=1,\ n_H=10^{20}$



Absorbed Plasma : $kT=2,\ n_H=10^{20}$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