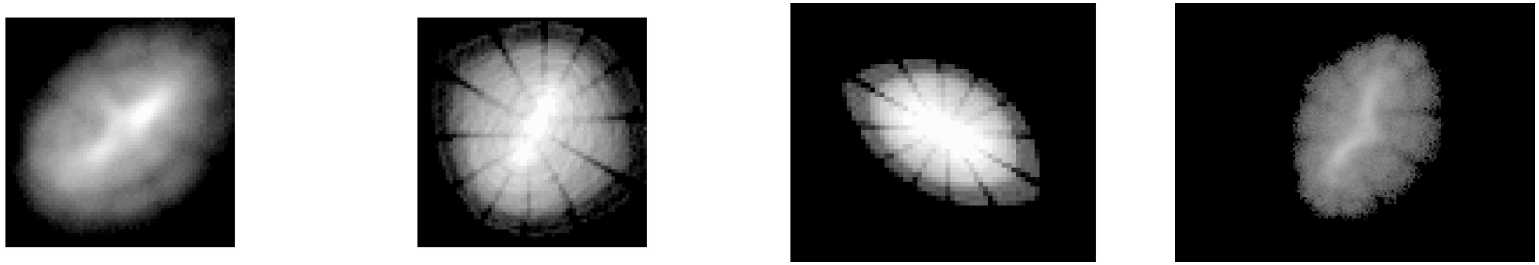
# A Parameterization of the Chandra Point Spread Function

Christopher L. Allen, Diab Jerius, and Terry Gaetz
SAO

27 Oct 2003

# Overview

The off-axis *Chandra* PSF is a complex function of source position and energy:



It is roughly elliptical, more so as off-axis angle increases.

Elliptical fits to simulations give a reference for comparison with real data.

We parameterize the PSF with Enclosed Count Fraction (ECF) values, the fraction of counts inside a given ellipse.

A grid of the parameters from the ellipses can be used as a guide in reducing data; users won't need to run lengthy raytraces to:

- remove off-axis point sources from background
- distinguish point sources from extended sources at large off-axis angles.

# Simulations

Off-axis point sources were simulated with the following parameters:

- energy: 0.277, 1, 1.5, 3, 4.6, 6.4, and 8 keV
- off-axis angle: A range of angles from $0'$ to $20'$
- azimuthal angle: 20+ angles, covering the detector plane

We simulated detection with an HRC-I model. The HRC-I has several properties that simplify our analysis and help us test the models better:

- It is perpendicular to the focal plane
- It is not a tiled detector (*e.g.* ACIS)
- It is the largest detector

To parameterize the simulations, we:

- fit ellipses to the simulation data
- calculated the ellipses' ECF:
  - $ECF = C_{ellipse}/C_{tot}$, where
  - $C_{ellipse}$ is the counts inside the ellipse
  - $C_{tot}$ is the total number of counts in the simulation

Note that we parameterize the SAOsac model, *not* actual observations:

- in general, the data do not have enough statistics and breadth in focal plane or energy coverage.
- We validate the parameterization against real data for consistency.

# Ellipse Fitting Theory

We use a moment-fitting technique to create the fitted ellipses. The $0^{th}$, $1^{st}$, and $2^{nd}$ moments of the data (f (x,y)) are calculated:

$$\mu_{ij} = \int\int dxdy\, f(x,y)x^i y^j \Longrightarrow \sum_x \sum_y f(x,y)x^i y^j.$$

and the parameters of the fitted ellipse (center $\overline{x}$ and $\overline{y}$, axes $a$ and $b$, and position angle $\phi$) are determined from the moments:

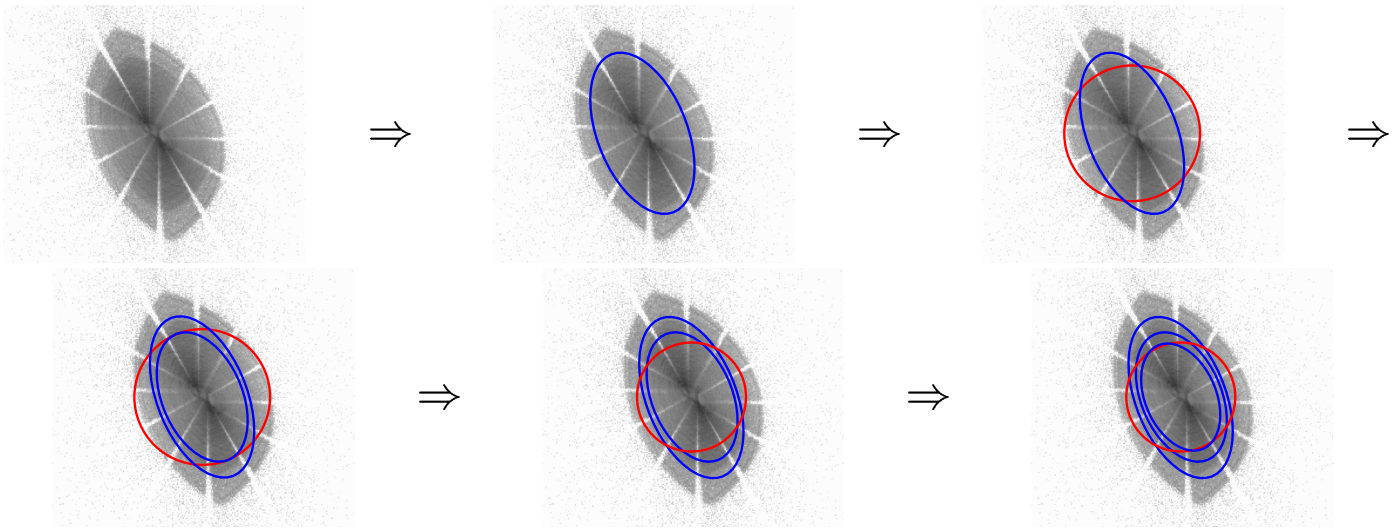
$$\overline{x} = \mu_{10}/\mu_{00}, \qquad \overline{y} = \mu_{01}/\mu_{00}, \qquad \phi = (1/2)tan^{-1}\left(\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}\right),$$

$$a = \left(\frac{\mu_{20}+\mu_{02}+[(\mu_{20}-\mu_{02})^2+4\mu_{11}^2]^{1/2}}{\mu_{00}/2}\right)^{1/2},$$

$$b = \left(\frac{\mu_{20}+\mu_{02}-[(\mu_{20}-\mu_{02})^2+4\mu_{11}^2]^{1/2}}{\mu_{00}/2}\right)^{1/2}.$$

# Ellipse Fitting Technique

- An initial ellipse was fit to the data subset containing the central 95% of the flux
  - Fitting this subset removes scattered photons, improves core fit
- New ellipses were made by fitting the subset of rays that were less than $0.8a$ from the previous ellipse's center:



We repeated this process until a useful number of ellipses was produced or the fit stuck.

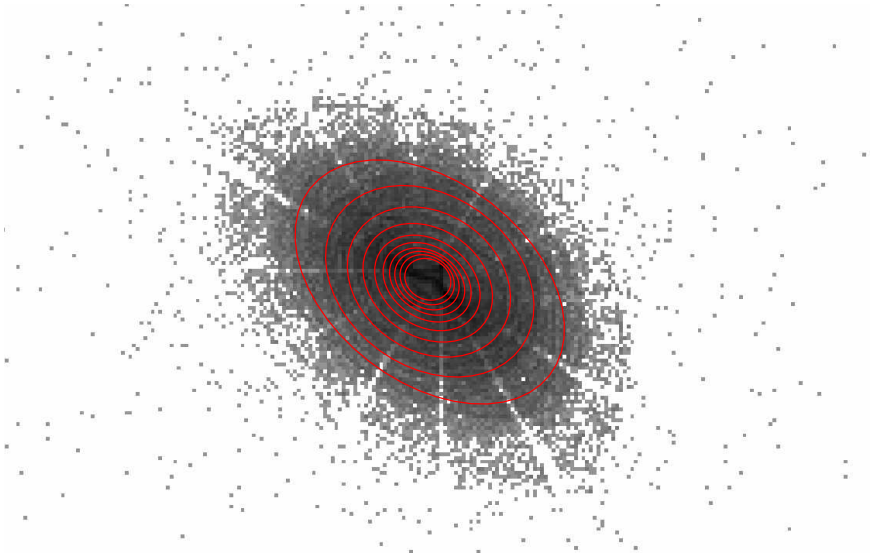
The fitting region shrinks after each fit, so each subsequent fit is a smaller ellipse.

Ellipses that intersected with older ellipses were rejected:

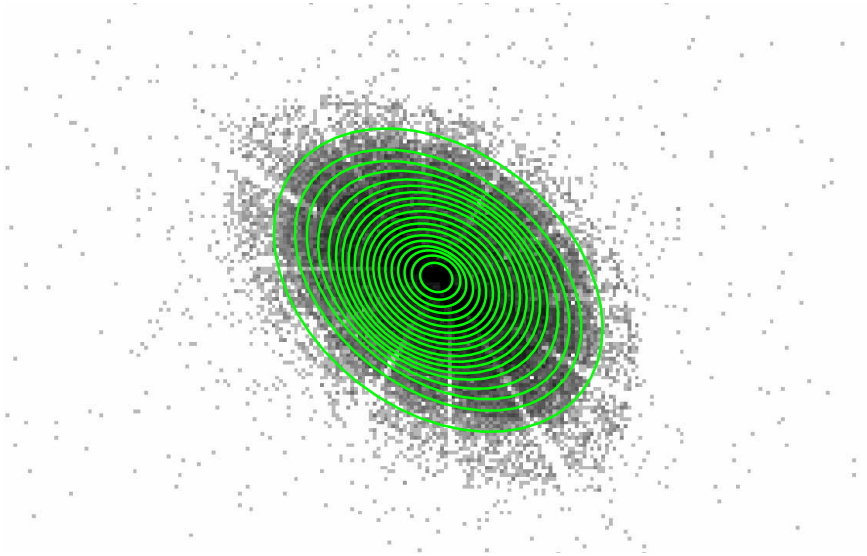
- ECF must be useful to user: this is a simple, non-physical parameterization
- Intersecting ellipses don't make sense

# Ellipse Fitting Technique, con't

The resulting set of ellipses is somewhat sparse:



So we generate ellipses for specific ECF values by interpolating or extrapolating from existing ellipses:



# Comparison with Chandra Observations

We selected 17 point sources observed by *Chandra* with HRC-I to compare to the simulations.

- We simulated the observations in azimuthal and off-axis angle,
- We were unsure of the energy of the objects because the HRC-I has no energy resolution.
  - We assumed the objects were low-energy (most sources are) and simulated them as monochromatic 1.5 keV objects (the HRMA effective area is fairly flat in that energy range).

We applied two transformations to the ellipses before applying them to the observations:

- The shift between image centroid and ellipse center was preserved
- The ellipses were rotated about the observation's centroid by the spacecraft roll value.

After these transformations were applied, the counts inside each of the simulation's ellipses were measured on the (background-subtracted) observation.

# Comparison with Chandra Observations, con't

The ECF value from the simulation's 85% ellipse was used as a baseline to normalize both the simulated ECF results and those from the Chandra observations.

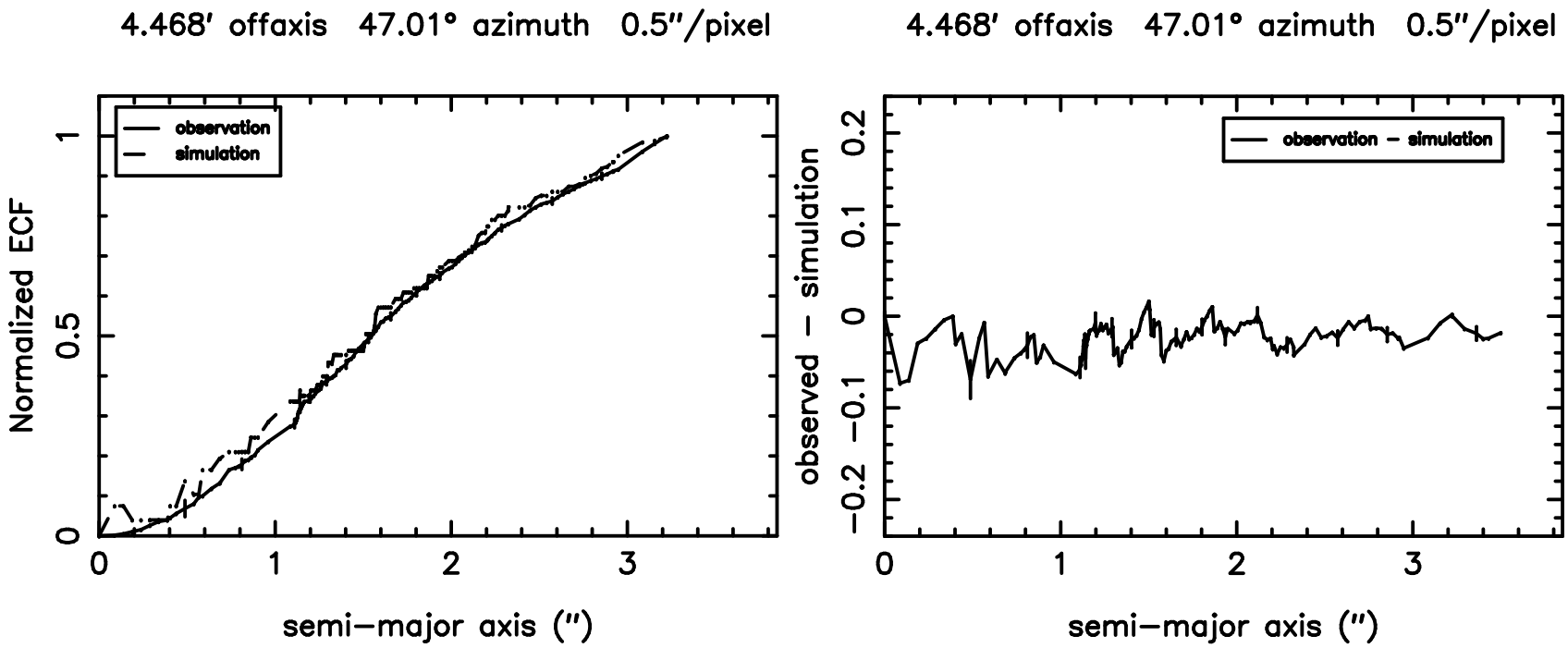
The 85% ECF ellipse was chosen as our metric because of:

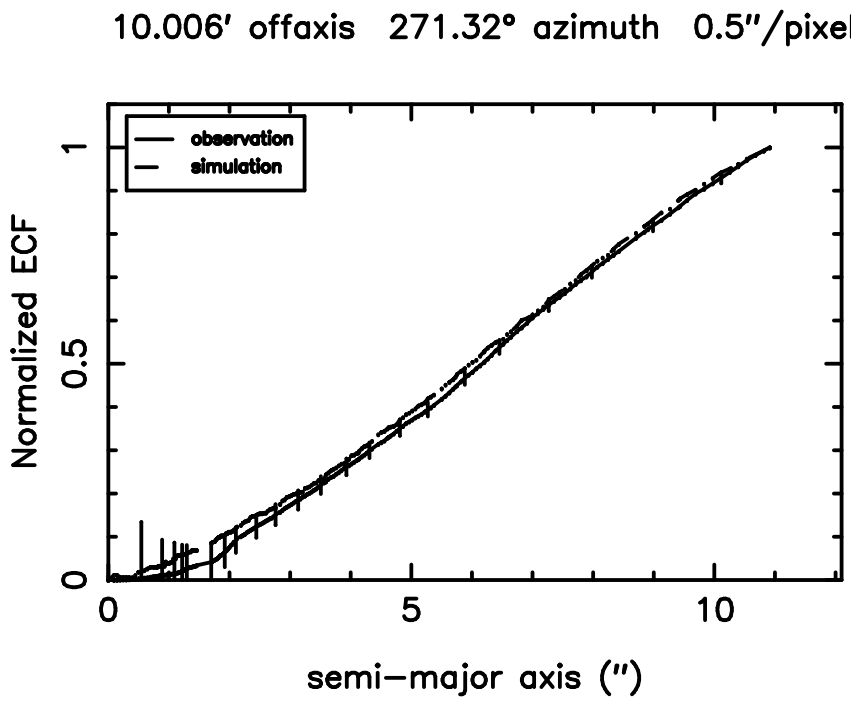
- Uncertainty of simulation correctness out in PSF wings, and
- Smaller background-subtraction uncertainty than at larger ECFs

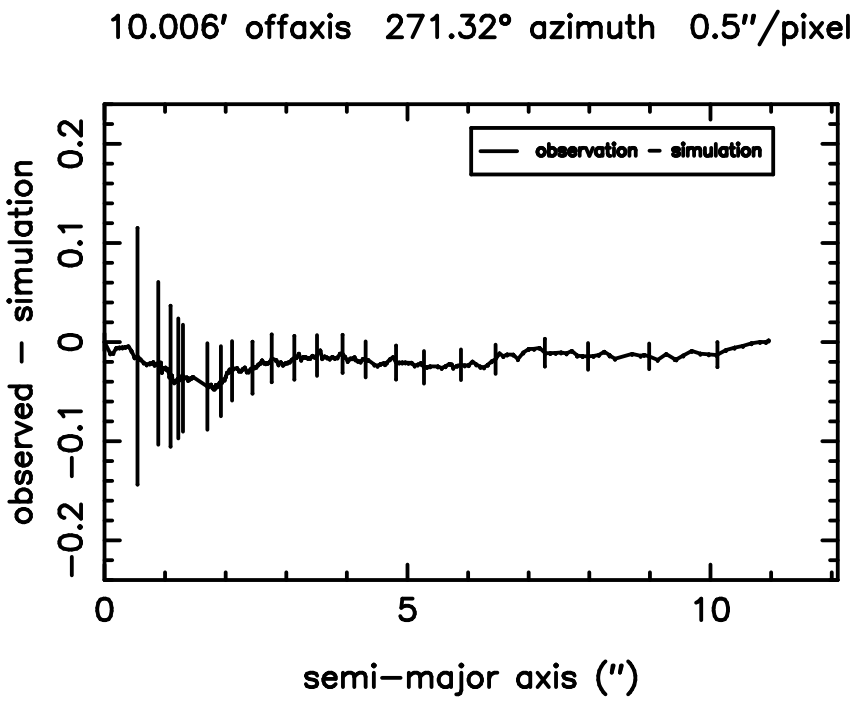
The ECF values for both the simulations and the Chandra observations were normalized by this value, and then compared.

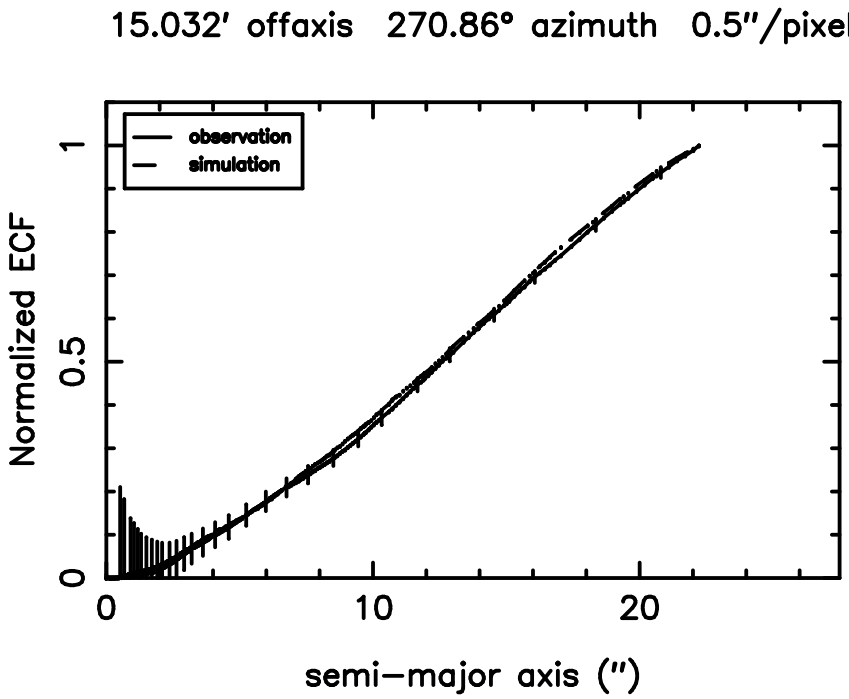
- The simulations and the observations were generally in agreement, within Poisson uncertainties, below the 85% normalization baseline.
- Error bars were usually below 5%, except in the core of PSFs at large off-axis angles, where there were low counts.

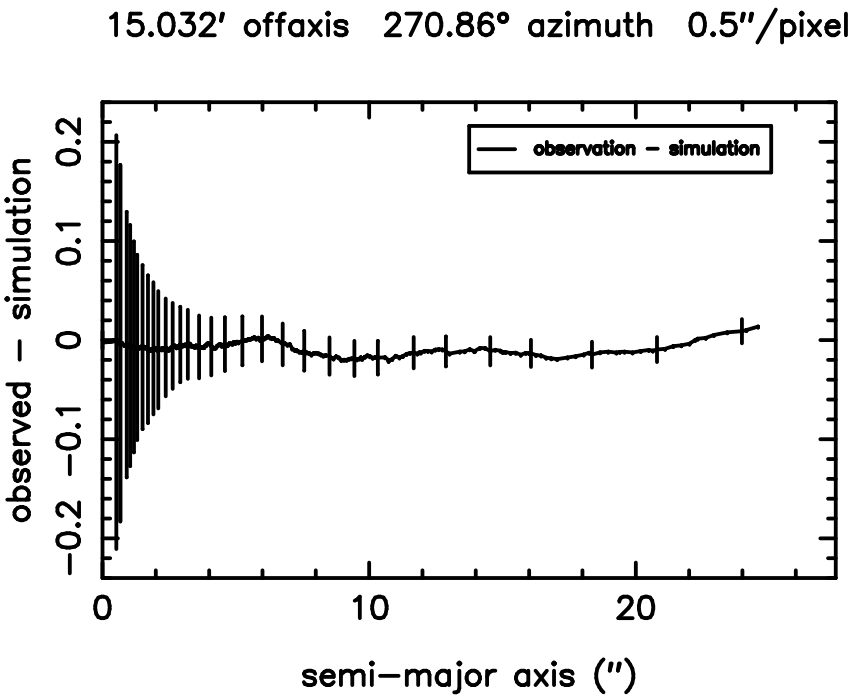
Some representative comparisons, with residuals:

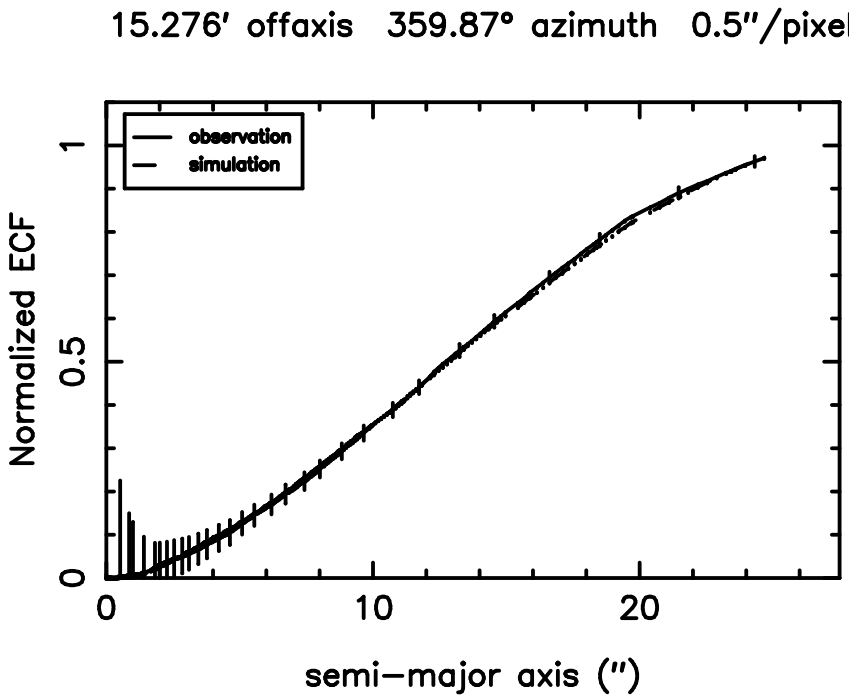

4.468′ offaxis 47.01° azimuth 0.5″/pixel
observation
simulation
Normalized ECF
0
0.5
1
0
1
2
3
semi−major axis (″)
4.468′ offaxis 47.01° azimuth 0.5″/pixel
observation − simulation
observed − simulation
−0.2
−0.1
0
0.1
0.2
0
1
2
3
semi−major axis (″)

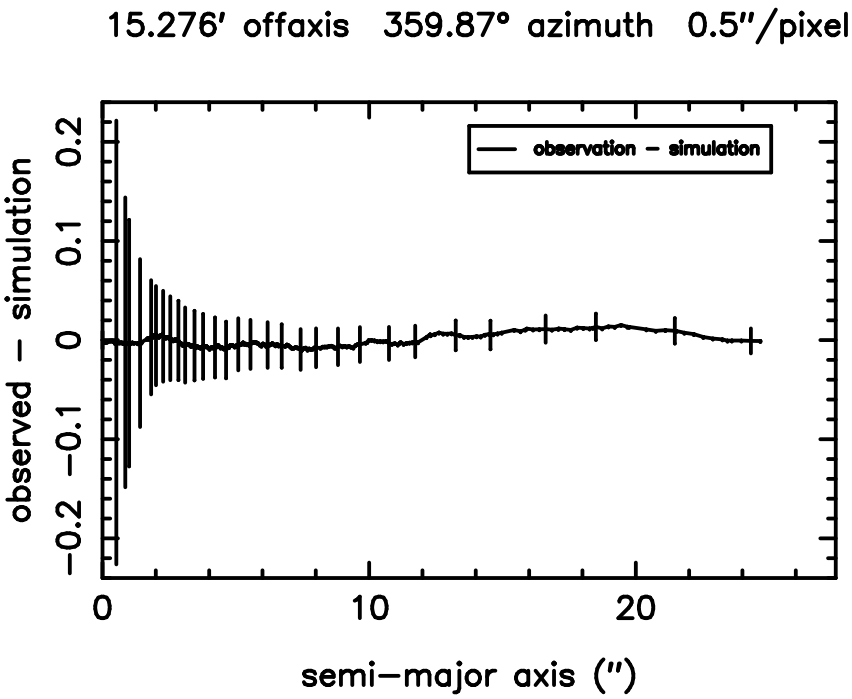

10.006′ offaxis 271.32° azimuth 0.5″/pixel
observation
simulation
Normalized ECF
0
0.5
1
0
5
10
semi−major axis (″)


10.006′ offaxis 271.32° azimuth 0.5″/pixel
observation − simulation
observed − simulation
−0.2
−0.1
0
0.1
0.2
0
5
10
semi−major axis (″)


15.032′ offaxis 270.86° azimuth 0.5″/pixel
observation
simulation
Normalized ECF
0
0.5
1
0
10
20
semi−major axis (″)

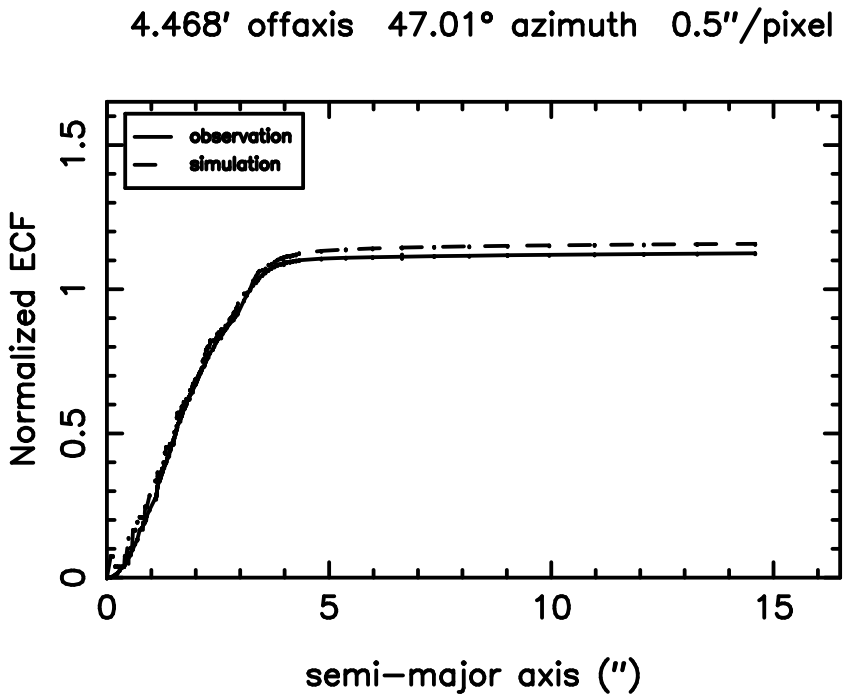

15.032′ offaxis 270.86° azimuth 0.5″/pixel
observation − simulation
observed − simulation
−0.2
−0.1
0
0.1
0.2
0
10
20
semi−major axis (″)

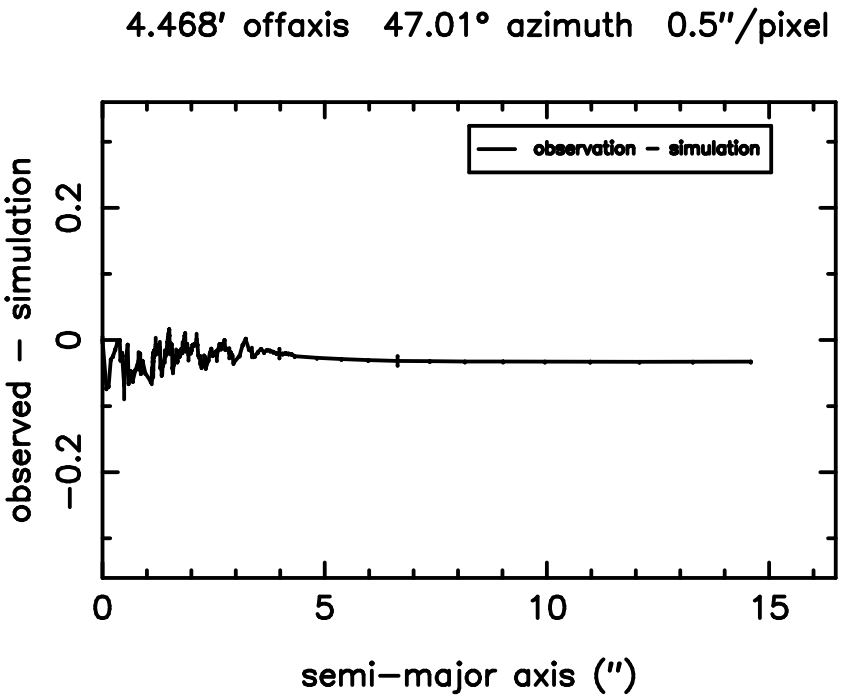

15.276′ offaxis 359.87° azimuth 0.5″/pixel
observation
simulation
Normalized ECF
0
0.5
1
0
10
20
semi−major axis (″)

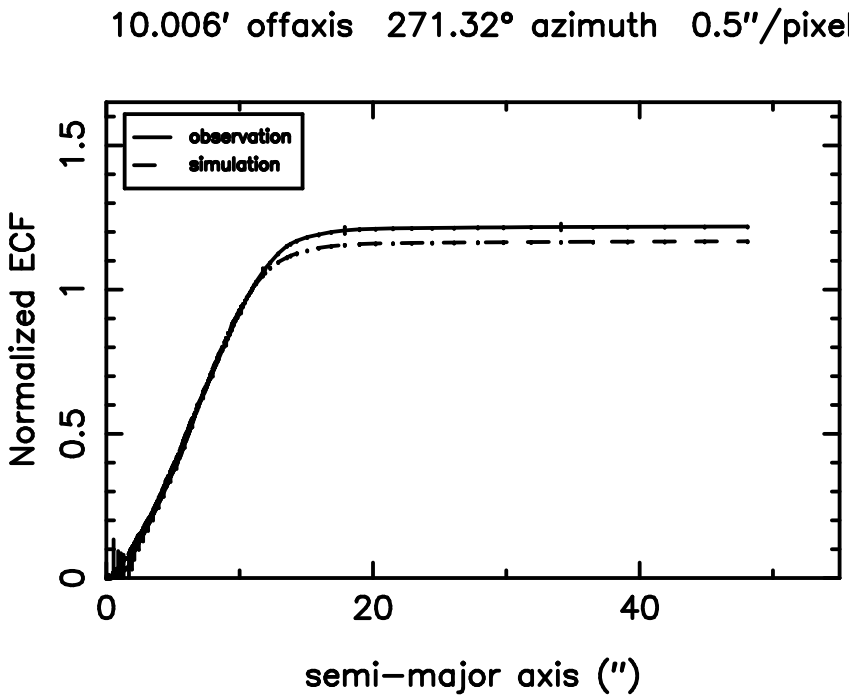

15.276′ offaxis 359.87° azimuth 0.5″/pixel
observation − simulation
observed − simulation
−0.2
−0.1
0
0.1
0.2
0
10
20
semi−major axis (″)

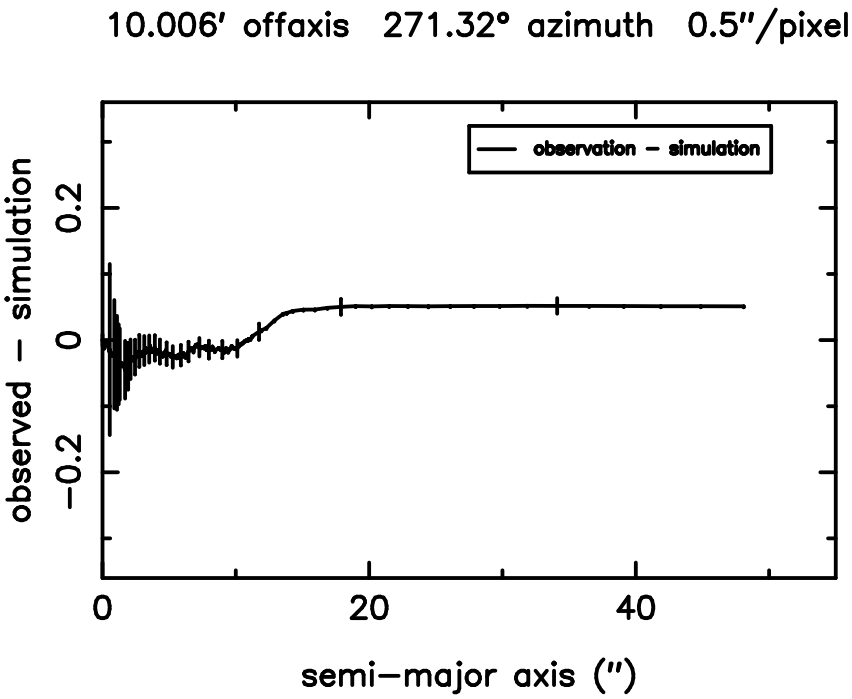
# Problems

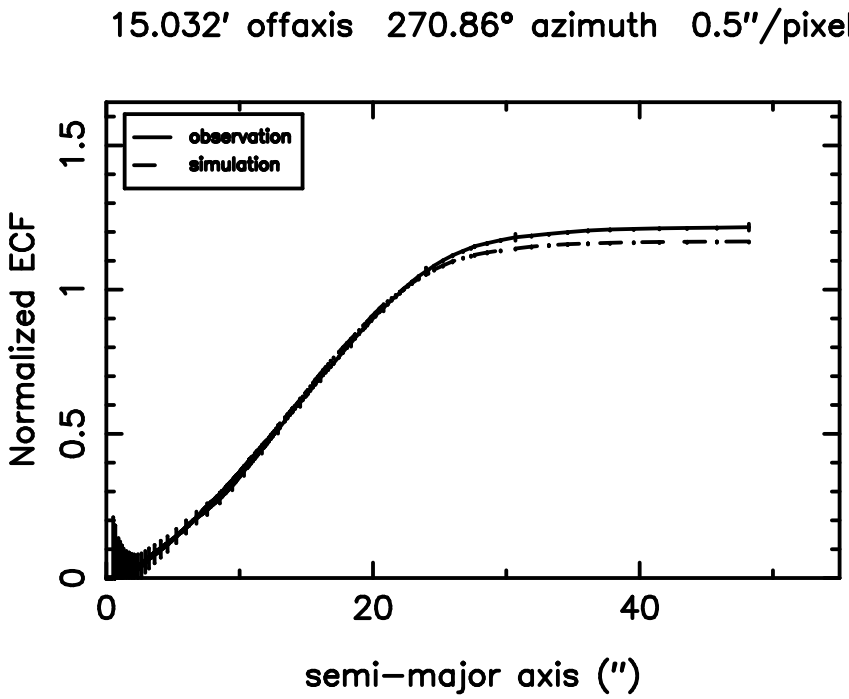
- Background subtraction needs to be refined.
- Simulations may be misestimating the core, as shown by the comparisons above the 85% normalization baseline.

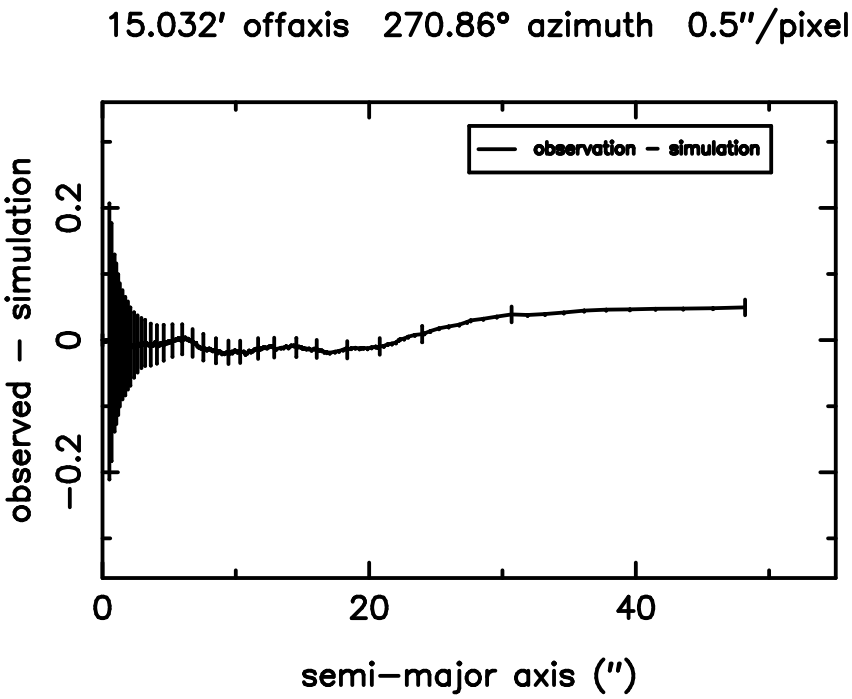


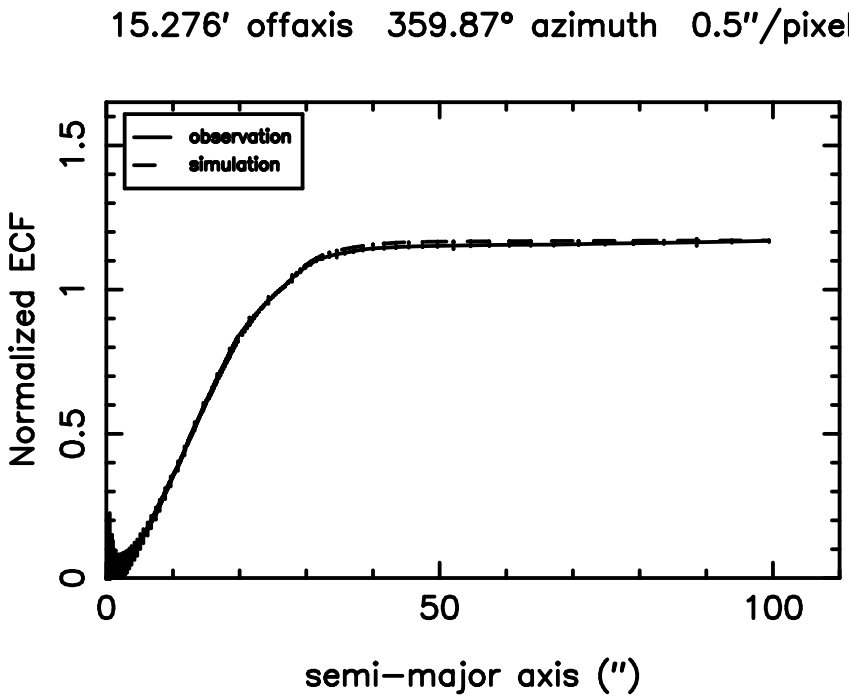



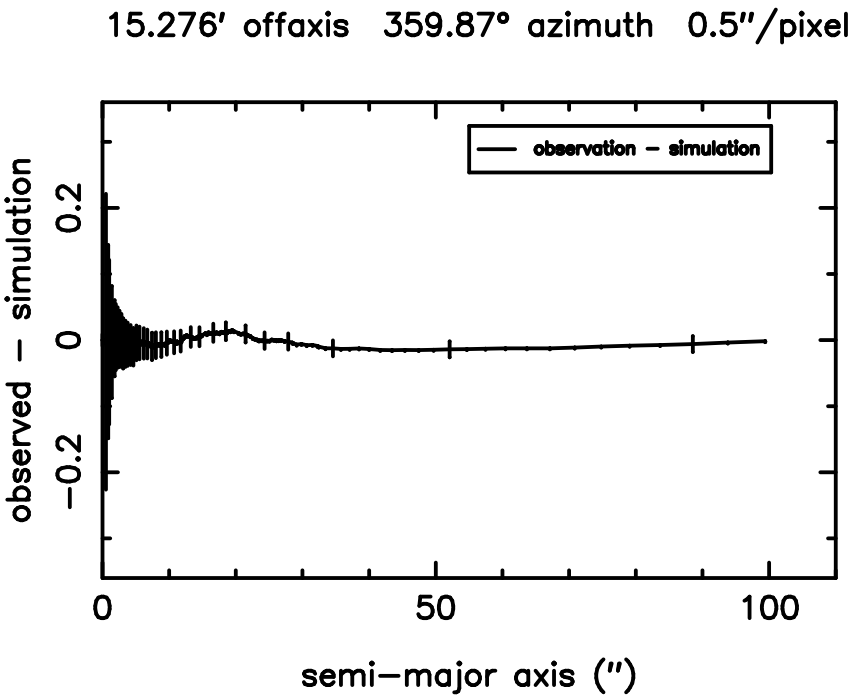

10.006′ offaxis 271.32° azimuth 0.5″/pixel
observation
simulation
Normalized ECF
0
0.5
1
1.5
0
20
40
semi−major axis (″)


10.006′ offaxis 271.32° azimuth 0.5″/pixel
observation − simulation
observed − simulation
−0.2
0
0.2
0
20
40
semi−major axis (″)


15.032′ offaxis 270.86° azimuth 0.5″/pixel
observation
simulation
Normalized ECF
0
0.5
1
1.5
0
20
40
semi−major axis (″)


15.032′ offaxis 270.86° azimuth 0.5″/pixel
observation − simulation
observed − simulation
−0.2
0
0.2
0
20
40
semi−major axis (″)


15.276′ offaxis 359.87° azimuth 0.5″/pixel
observation
simulation
Normalized ECF
0
0.5
1
1.5
0
50
100
semi−major axis (″)


15.276′ offaxis 359.87° azimuth 0.5″/pixel
observation − simulation
observed − simulation
−0.2
0
0.2
0
50
100
semi−major axis (″)

# Data Products

The data products from this work are a set of tables describing the ellipse parameters:

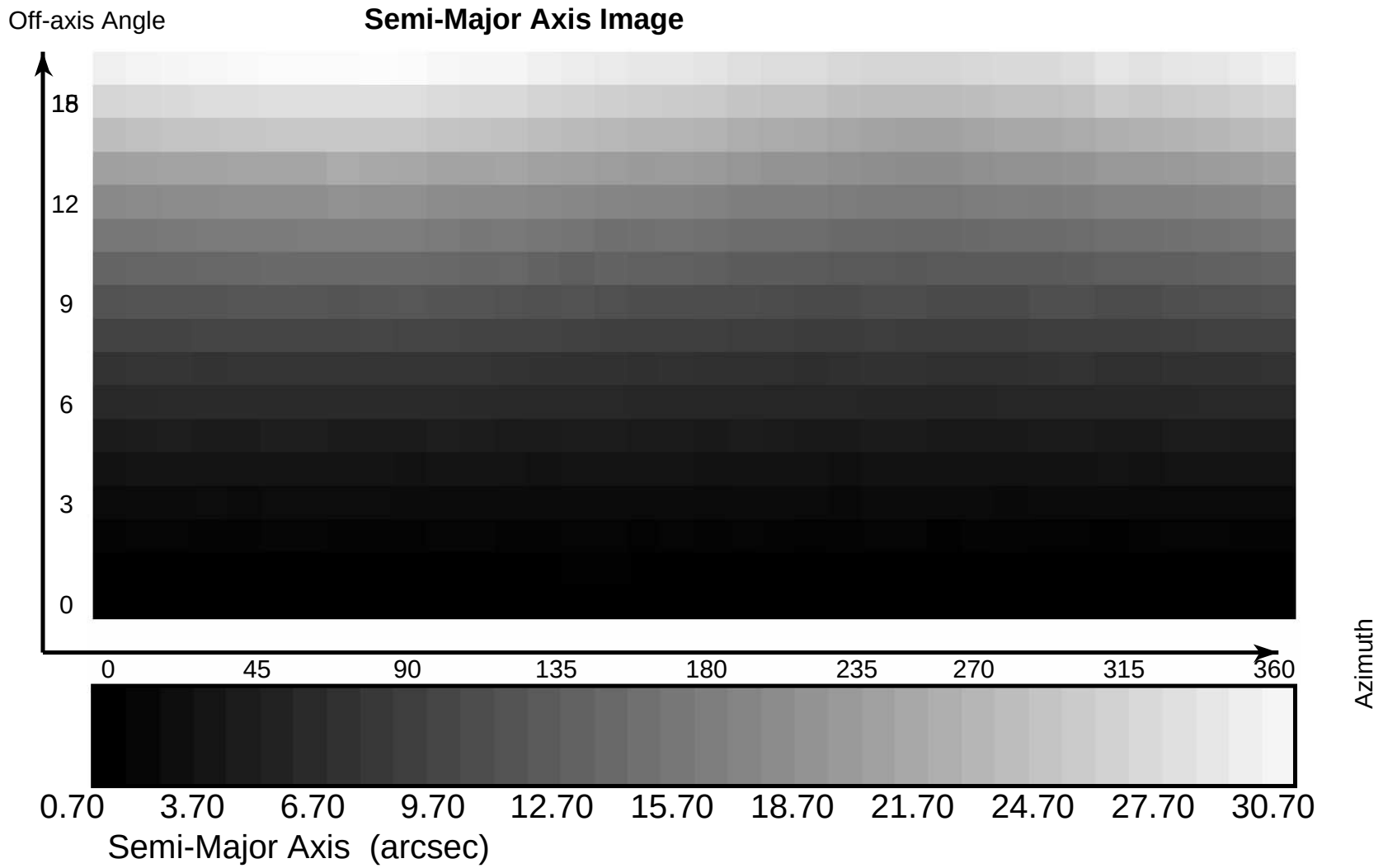
- $a$
- $\phi$
- $e$

as a function of:

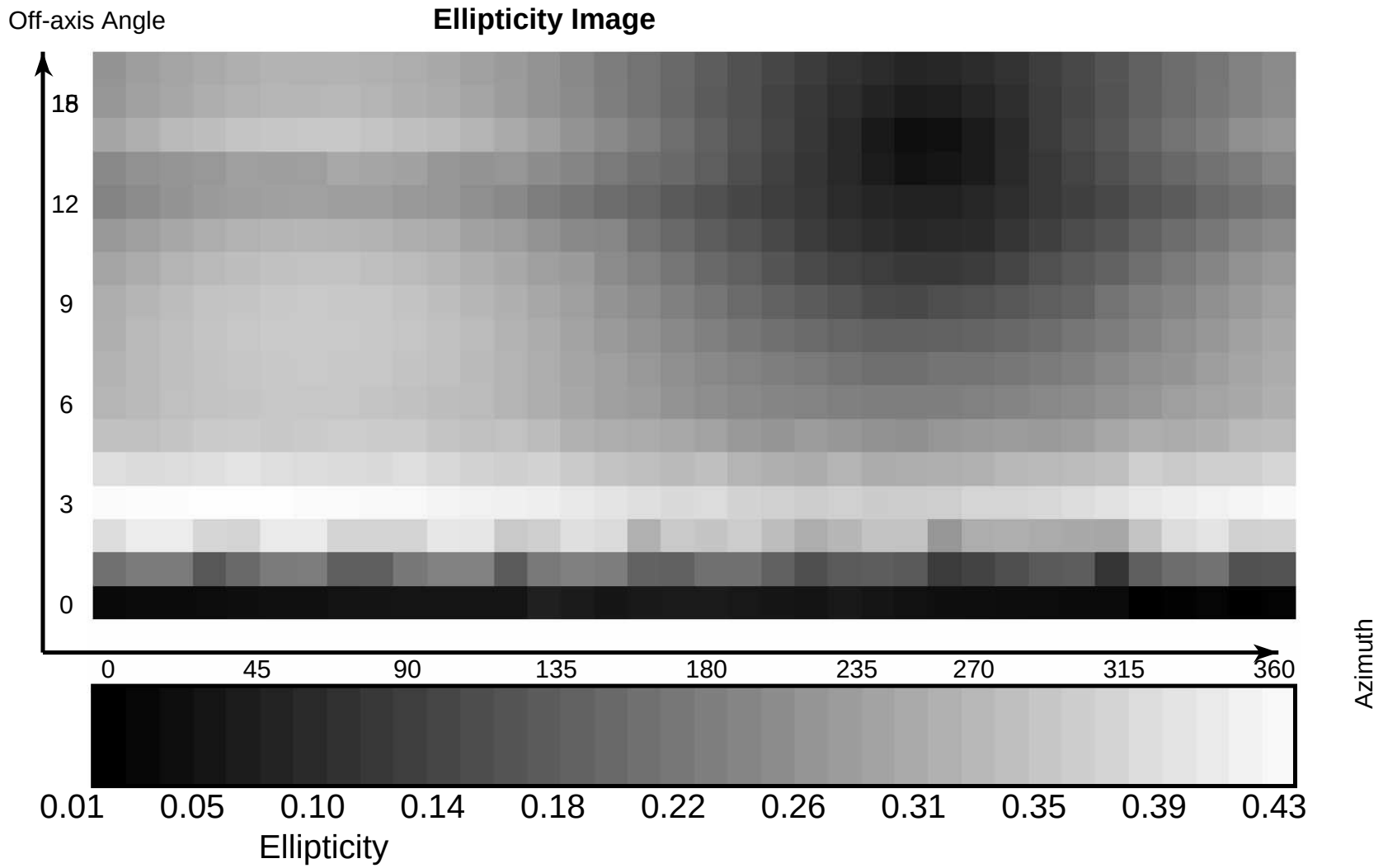
- ECF
- off-axis angle
- focal position

Additional representations of the data are available, in imagemap and plot form:

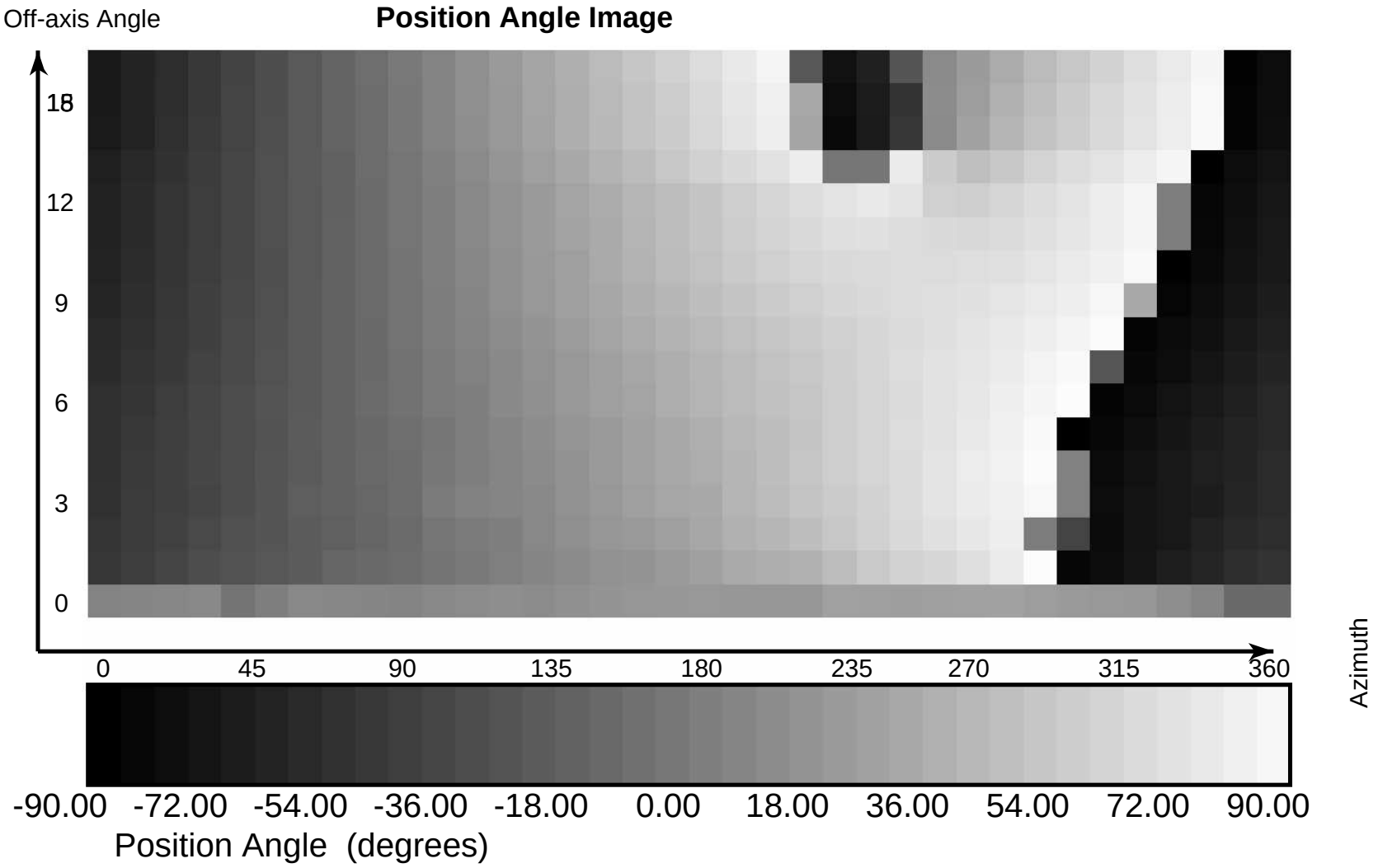
Imagemap of $a$ for 85% ECF ellipses:



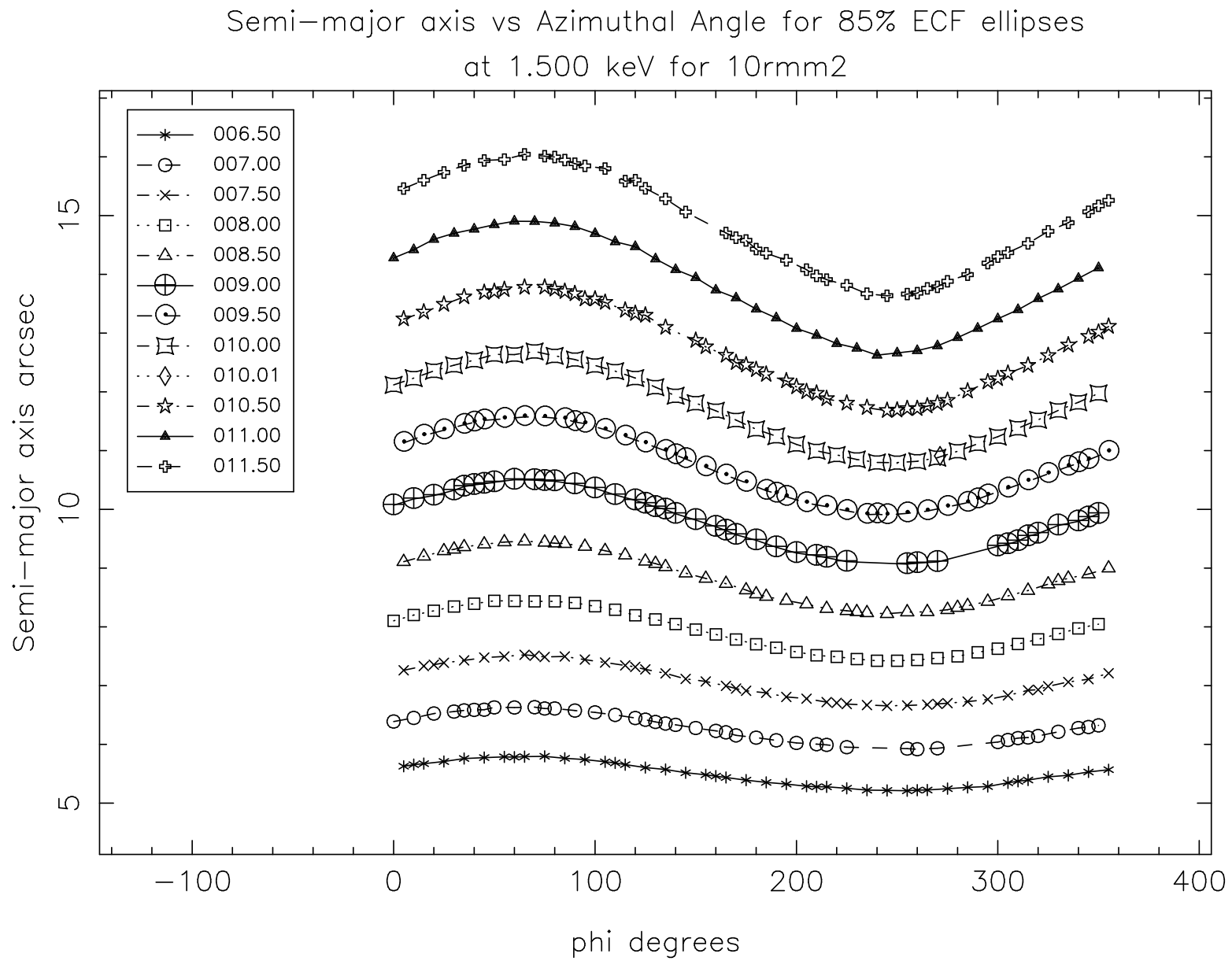
Imagemap of *e* for 85% ECF ellipses:



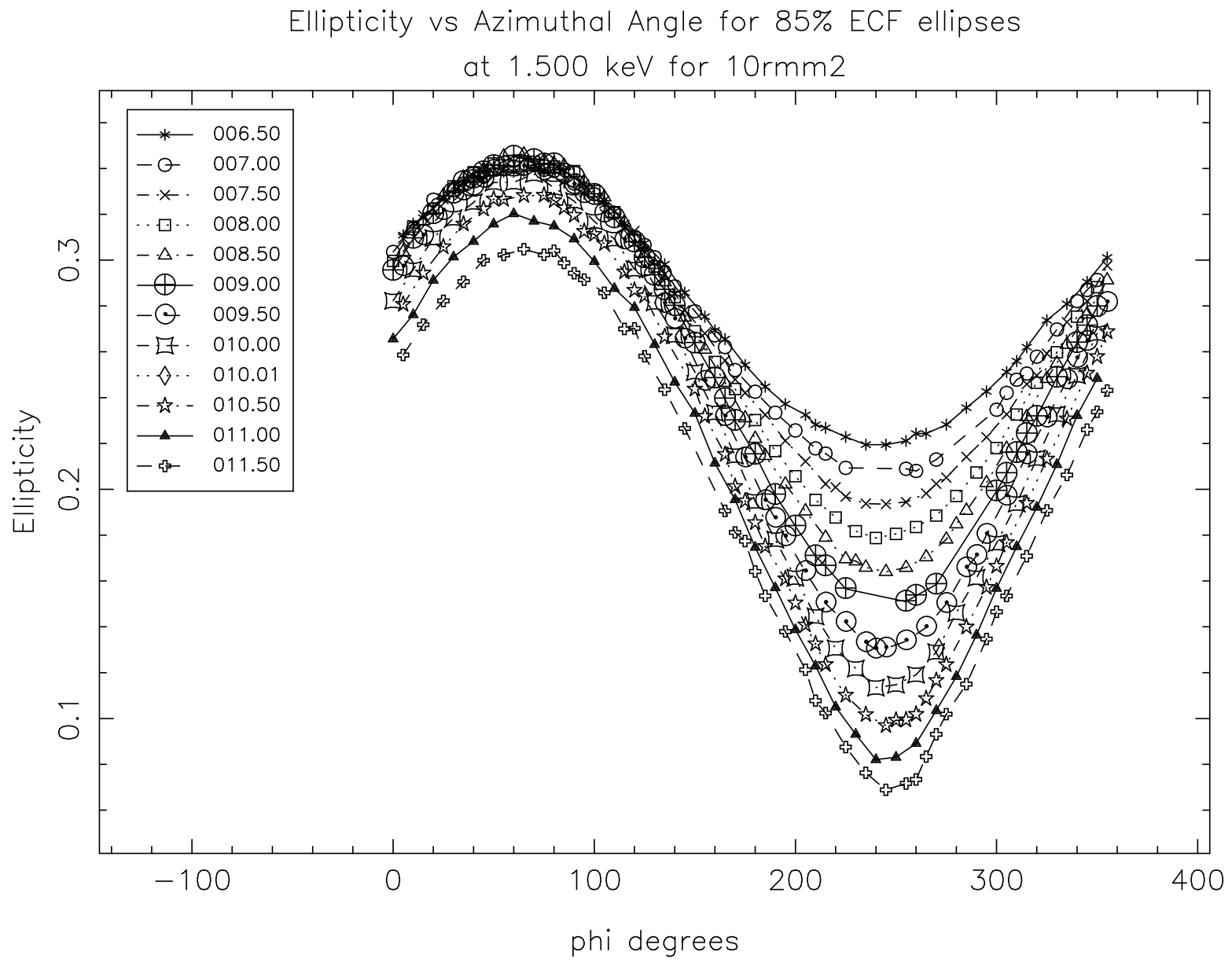
Imagemap of $\phi$ for 85% ECF ellipses:



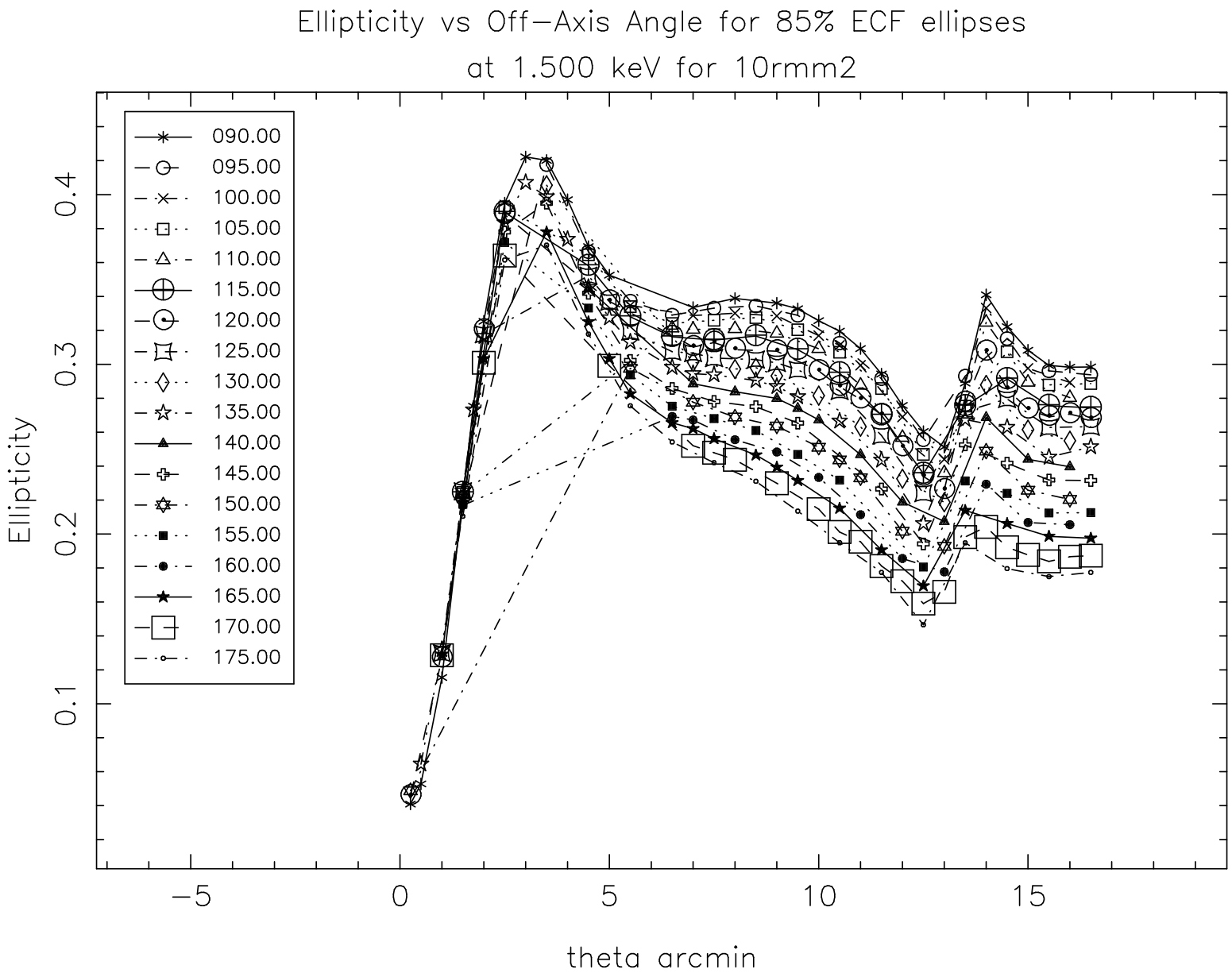
*a* vs. azimuthal angle for 1.5 keV, at a selection of off-axis angles:



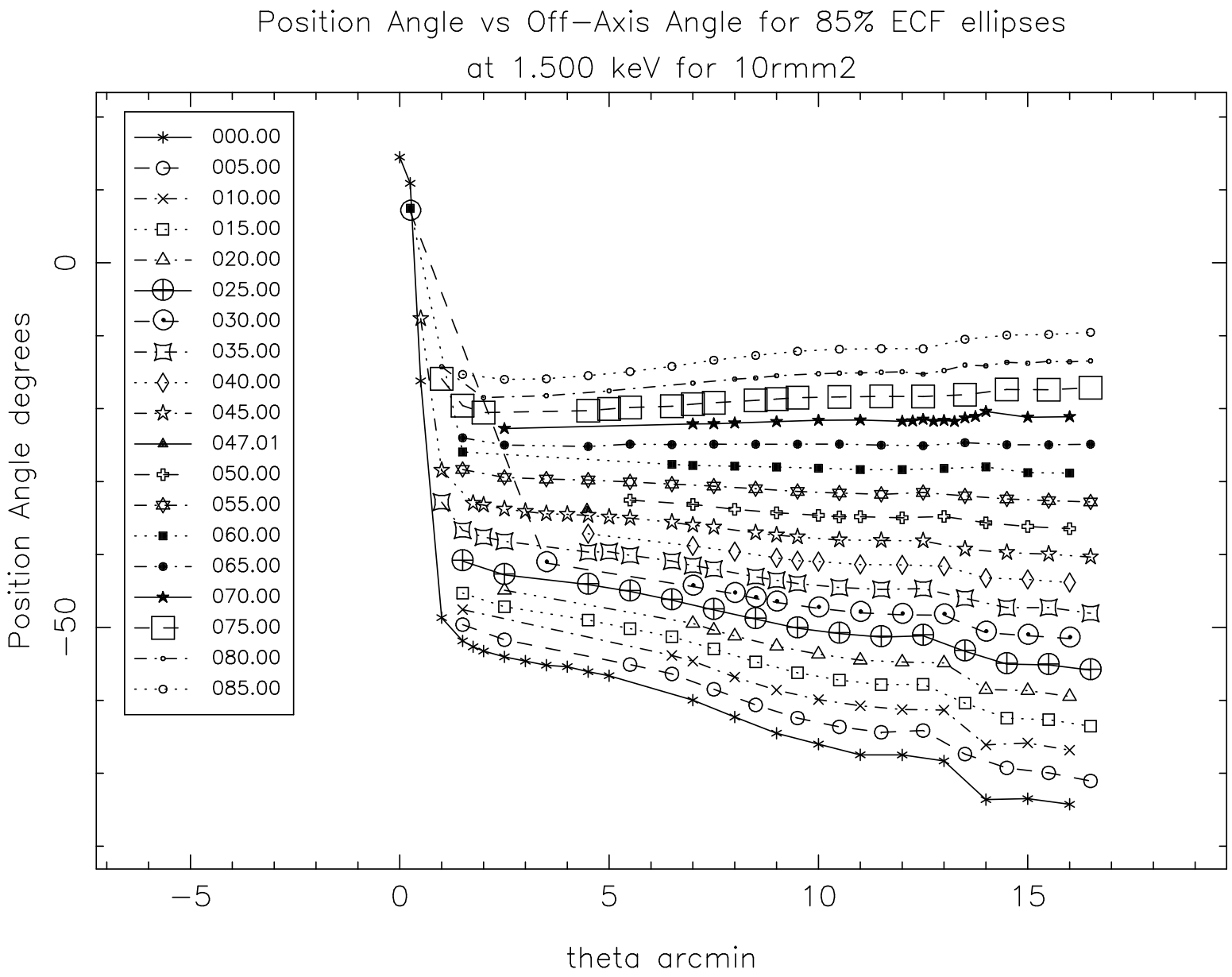
*e* vs. azimuthal angle for 1.5 keV, at a selection of off-axis angles:



*e* vs. off-axis angle for 1.5 keV, at a selection of azimuthal angles:



$\phi$ vs. azimuthal angle for 1.5 keV, at a selection of off-axis angles:



# Conclusions

- Products will be available on the web to users within two months.
- Trends in the data may be able to better-characterize mirror misalignment.
- Additional work needed to investigate comparison discrepancy above baseline.