ACIS Memo # PS–146

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To: ACIS Team
Subject: The branching ratio technique for the depletion depth measurement.
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1. Introduction

In a bunch of memos issued by *kcg* and *gyp* some years ago numerous techniques were suggested for determining a depletion depth of a CCD. All of the suggested techniques were very complicated and not too accurate. This memo describes the most simple, efficient and stable technique. It is not sensitive to the way the counts are counted (for example, reasonably small changes in thresholds do not effect the result), does not require any assumptions about the potential profile, and, probably, the most important, depletion depth can be extracted fron just one dataset of Fe^{55} data (no need to step through the range of voltages, for instance).

2. Description of the technique

This technique makes use of the following idea. Let us assume that CCD is illuminated with a monochromatic source of X-ray photons whose characteristic absorption length λ is smaller, but

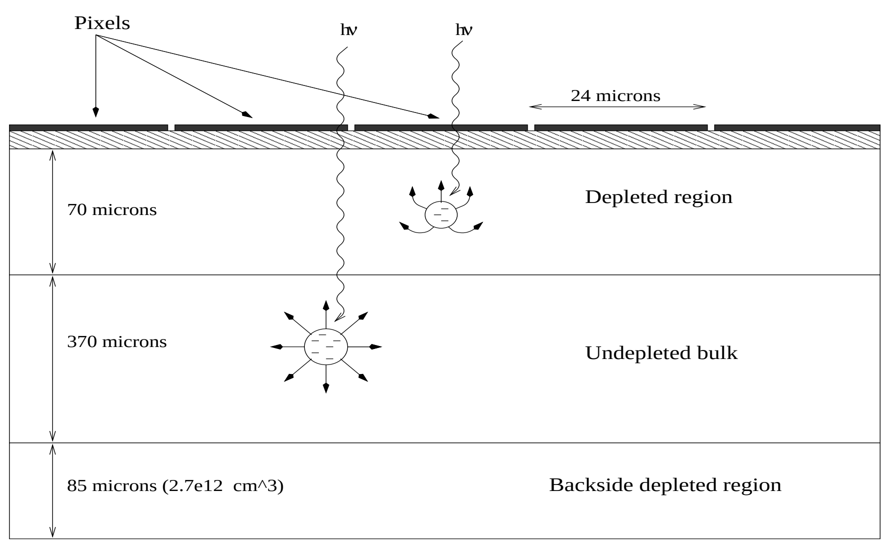

Figure 1: Schematic of the CCD depleted layers.

comparable to the depletion depth. If an interaction of the photon with Si occurres in the depleted region of the CCD, the created electron cloud will be drawn into the potential wells by the electric field and all the charge will be collected (see Fig. 1).

Depending on where relative to pixel boundaries the photon landed the resulting event can be either a single-pixel or a split one (can be either horizontally or vertically split, or grade 6), but the sum of the amplitudes of the pixels containing signal charge should account for all the charge generated by the photon, and, hence, be part of the main peak in the response histogram. On the other hand, if an interaction occurred in a neutral bulk of the semiconductor, the charge cloud diffuses more or less uniformly in all directions. Part of the cloud drifting towards the back side of the device will be lost. Electrons moving towards the front surface enter the depletion region and being pulled by an electric field end up in the potential wells of the CCD. Due to initial diffusion of charge the registered event in this case is a widespread multipixel cloud. According to the ASTRO-D grading scheme this is a grade 7 event and, since part of the initial charge is lost to the backside junction, the amplitude is lower than the peak energy even if all the pixels of the event are summed together.

If we denote N_d the number of interactions in the depleted region and N_{und} the number of events in the undepleted bulk, then, due to exponential distribution of the of the number of interacting photons as as a function of depth, a simple equation holds (see Fig. 2 for better understanding):

$$N_{und} = (N_d + N_{und}) \exp(-\frac{d_d}{\lambda}) \tag{1}$$

where d_d is the depletion depth of the device.

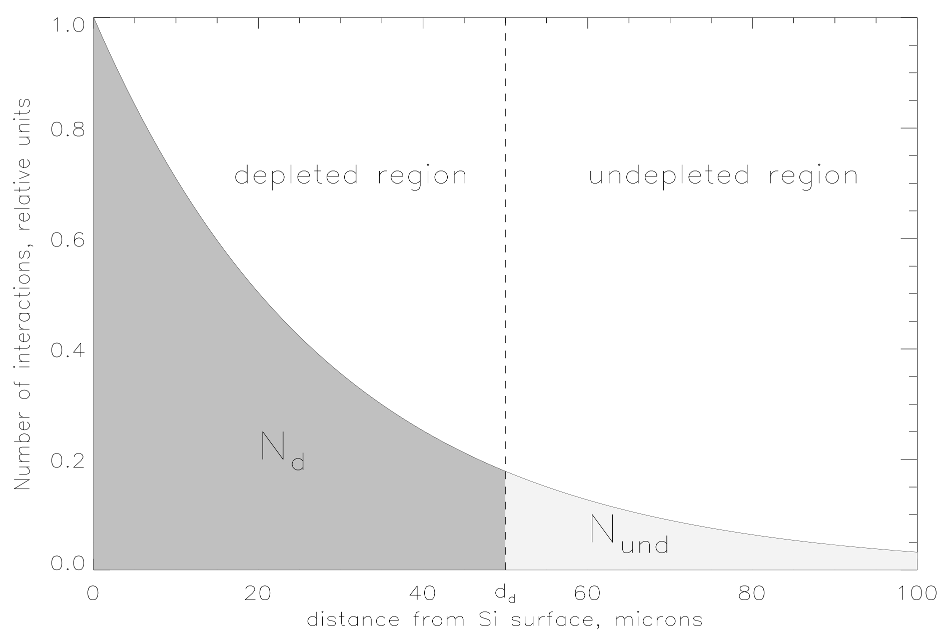

Figure 2: Exponential distribution of the number of interacting photons as a function of distance from the surface.

This implies the following algorithm of the depletion depth calculation.

1. Sum the the intensities of the grades 0,1,2,3,4,6 and determine what is the number of events N_d in the main peak. Usually I find the peak center E_c and width σ, and then count how many events are within $\pm 3\sigma$ interval from the peak center.

2. Count what is the total number of events N_{und} in the grade 7 below $E_c - 3\sigma$.

3. Calculate the depletion depth d_d according to the formula derived from (1):

$$d_d = \lambda \ln(\frac{N_d}{N_{und}} + 1) \tag{2}$$

The logarithmic function in the above equation makes the result very insensitive to small changes in both N_d and N_{und} caused by statistical uncertainties or, say, choosing different set of grades for the peak counts calculation. At the same time, small changes of the depletion depth cause changes in the N_d and N_{und} to go in the opposite directions (because the sum $N_d + N_{und} = const$, see Fig. 2), thus amplifying the change in the result. Sharp sensitivity of the result to the changes of the depletion depth has been confirmed in the experiment where the gate voltage was stepped from 0 to +10 Volts.

3. A typical example and possible sources of errors

3.1 Statistical errors

Here is an exemplary calculation of depletion depth for the ACIS flight device *w193c2*. The dataset */ohno/di/database/w193c2/butthead/fe55/30nov96/1713/* represents 416 frames of Fe^{55} X-rays taken in Butthead at -120^o C with standard imaging array voltages of +1 +11 Volts. The average count rate is 1110 $counts/sector/frame$. The calculation is made for the chip sector 2.

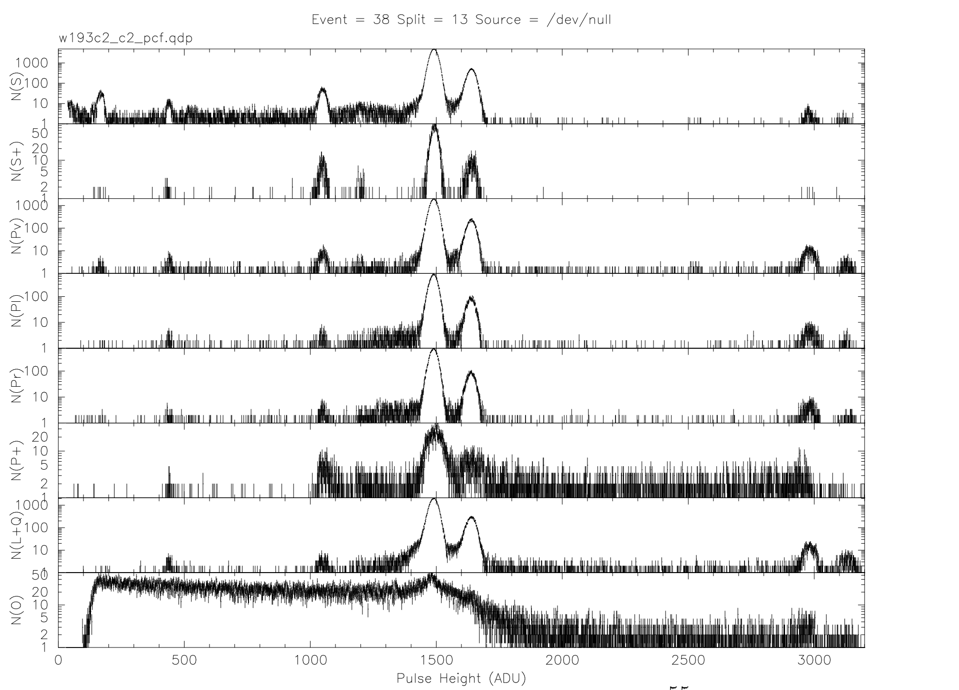

Figure 3: Histograms of different grades for Fe^{55} X-rays.

On Fig. 3 is shown a standard set of histograms of different grades. The number of counts in the K_α peak of combined grades 0 1 2 3 4 6 histogram within $\pm 3\sigma$ interval is 355719 ± 596 (an error is calculated as a square root of number of counts). Number of counts in grade 7 histogram below $E_c - 3\sigma$ is 33092 ± 182 (again assuming an error being square root, although, strictly speaking, it may not be the right estimate because this is not the gaussian distribution). This results in the depletion depth of 70.9 microns. Assuming the worst case of statistical errors going in the opposite directions we get the limits for d_d from 70.69 to 71.07 microns, which means that the statistical accuracy for the standard dataset is better than 0.3%.

3.2 K_β line inflicted error

There is a source of systematic error associated with the nonmonochromatic nature of the Fe^{55} radioactive source. Presence of the K_β line in the spectrum leads to underestimation of the depletion depth because the characteristic absorption length at 6.4 keV ($\lambda_\beta = 38.0$ microns) is bigger than at 5.89 keV ($\lambda_\alpha = 28.77$ microns) and, hence, K_β line produces bigger share of events in the undepleted bulk. To calculate the error introduced by the K_β line let us rewrite equation (1) separately for both lines:

$$N_{und\alpha} = (N_{d\alpha} + N_{und\alpha}) \exp(-\frac{d_d}{\lambda_\alpha}) \tag{3}$$

$$N_{und\beta} = (N_{d\beta} + N_{und\beta}) \exp(-\frac{d_d}{\lambda_\beta}) \tag{4}$$

Quantities $N_{d\alpha}$ and $N_{d\beta}$ can be determined from the experimental data as the number of counts in each of the corresponding peaks, whereas $N_{und\alpha}$ and $N_{und\beta}$ cannot be separated. But, since the sum $N_{und\alpha} + N_{und\beta}$ is known (it is number of counts in the grade 7 tail), equations (3) and (4) can be solved numerically after transforming them into the following:

$$\frac{N_{d\alpha}}{exp(\frac{d_d}{\lambda_\alpha}) - 1} + \frac{N_{d\beta}}{exp(\frac{d_d}{\lambda_\beta}) - 1} - (N_{und\alpha} + N_{und\beta}) = 0 \tag{5}$$

The program *~gyp/het/dep_Kbeta_corrected* implements Newton-Rafson method to solve (5). For the above example (number of counts in the K_β peak equals 45709) the corrected solution gives $d_d = 73.8$ microns, or, 4% increase of d_d.

3.3 Pileup

One other source of error can be pileup. It throws events out of the main peak into events with higher amplitude. The most simple first order correction would be to assume that all the events in the pileup peak (located at the energy $2E_c$) originates from two main peak photons landed on the top of each other, and that the "high energy tail" of grade 7 distribution (events to the right from the K_β peak) comes from one main peak photon and one grade 7 event piled up together. After adding the corresponding number of counts to the main peak and to the grade 7, the corrected depletion depth (using formula (2)) becomes 69.64 microns.

3.4 Interactions beyond detection limit

Some photons interact so deep in the bulk of silicon that the portion of charge collected in one pixel is too small to be detected. An event will not be registered by the data analysis software if the charge in each pixel falls below the event threshold. To determine the distance from the depletion region border beyond which the charge will stay undetected I used Steve Jones' code which implements solution of diffusion equation as described in Hopkinson's paper. Solid line on Fig. 4 shows charge collected in the pixel for 5.89 keV photon stopped in the field free region as a function of distance from the cloud center to the depletion region boundary. A dotted line in the plot shows the same thing calculated on the simple assumption that charge diffuses uniformly in all directions. This approach is attractive since the result can be expressed by a single analytical formula. In this case charge Q_{pix} collected in one pixel is proportional to the solid angle under which the pixel is seen from the center of the cloud. Solving the stereometry quiz in the case of

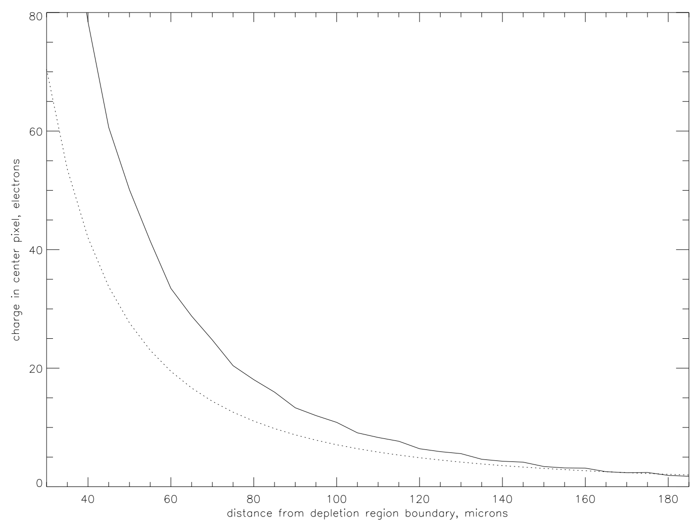

Figure 4: Charge collected in the center pixel from the Fe^{55} photon landing in the neutral bulk as a function of distance from depletion region boundary.

cloud center located at the pixel center line results in the following:

$$Q_{pix} = \frac{Q_{tot}}{4\pi}(8\arcsin\sqrt{\frac{2{d_c}^2 + a^2}{4{d_c}^2 + a^2}} - 2\pi) \tag{6}$$

where a is a pixel size, d_c is the distance from the center of the charge cloud to the depletion region border, Q_{tot} is total electron charge created by the photon. As expected, both curves in Fig. 4 converge at large distances, when the influence of the boundary becomes negligible. At small d_c simplified approach results in the collected charge approximately factor of 2 smaller than the full solution, which also makes sense. In the middle range of distances smaller than 140 microns, though, the difference is surprisingly big to justify the use of the simple approach.

For the data analyzed above event threshold is 38 ADU, or, multiplied by gain, 35 electrons. From the above plot (solid line) one can determine critical distance $d_c = 59$ microns beyond which a photon is not detected. It means that 1.1% of all the events stay undetected (the ones that are deeper than $70.9 + 59$ microns from surface) instead of being counted as grade 7 events.

In order to calculate the correction for the depletion depth introduced by the undetected photons the equation (1) should be modified into

$$N_{und} + N_{lost} = (N_d + N_{und} + N_{lost})\exp(-\frac{d_d}{\lambda}) \tag{7}$$

and supplemented by a similar equation for the region beyond the depletion boundary:

$$N_{lost} = (N_{und} + N_{lost})\exp(-\frac{d_c}{\lambda}) \tag{8}$$

where we introduced N_{lost} – number of interactions beyond the detection border located at the distance $d_d + d_c$ from the silicon surface.

These two can be converted into the corrected formula for the depletion depth

$$d_d = \lambda\ln(\frac{N_d}{N_{und}(\frac{1}{1-\exp(-\frac{d_c}{\lambda})})} + 1) \tag{9}$$

For the above analyzed example the corrected depletion depth according to (9) is 67.3 microns, which constitutes 5% correction. It is clear from this analysis that reducing event threshold will significantly improve the accuracy of the original formula (2).

3.5 Summary of error estimates

The conclusion is that statistical errors for the depletion depth determination using standard set of Fe^{55} frames are negligible (less than 0.3%). The biggest sources of systematic errors are K_β line photons and photons interacting deep in the bulk of silicon. Corrections from those two factors go in the opposite directions and tend to compensate each other because K_β photons increase number of events counted as grade 7, whereas undetected ones diminish this number. For the analyzed example they almost entirely cancelled each other, although, undetected photons seem to pull d_d down a little stronger. As a result, calculation made with the simple formula (2) seem to overestimate depletion depth by about 2% due to pileup and the detection limit. To get more accurate result a very detailed modeling is necessary.