The Role of GR and Tidal Effects in Dense Stellar Systems

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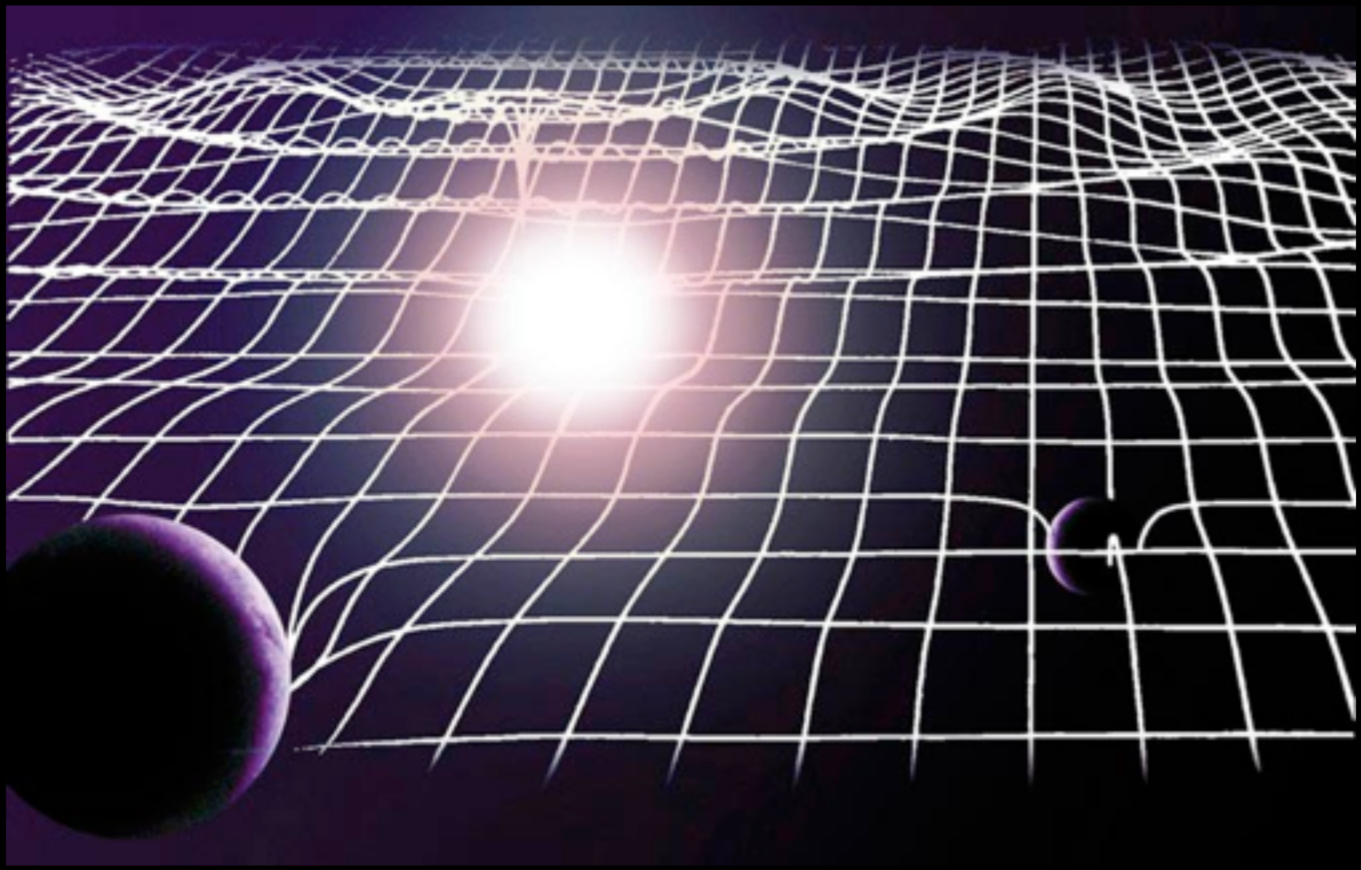


Modeling

Full N-body
- very expensive (for example Nbody6)
- Impossible to resolve rare outcomes

Monte Carlo Technics
- very fast
- currently used to calculate LIGO rates
- No tides - and maybe not even GR

Our approach:

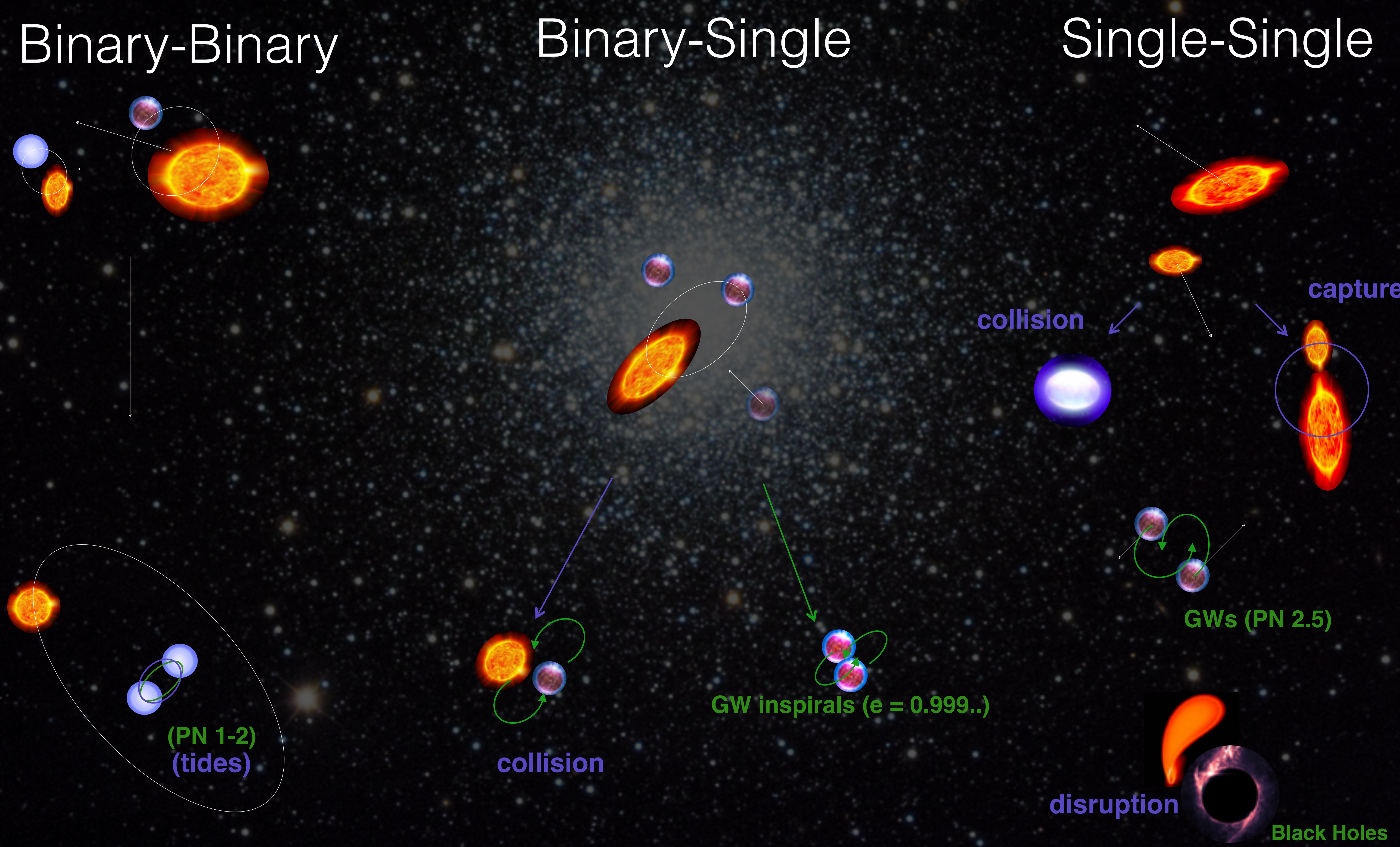
- First to try understand the role of tides and GR for individual interaction channels.

- Later implement in full cluster codes

Interaction Channels:

Include: GR

Include: Tides

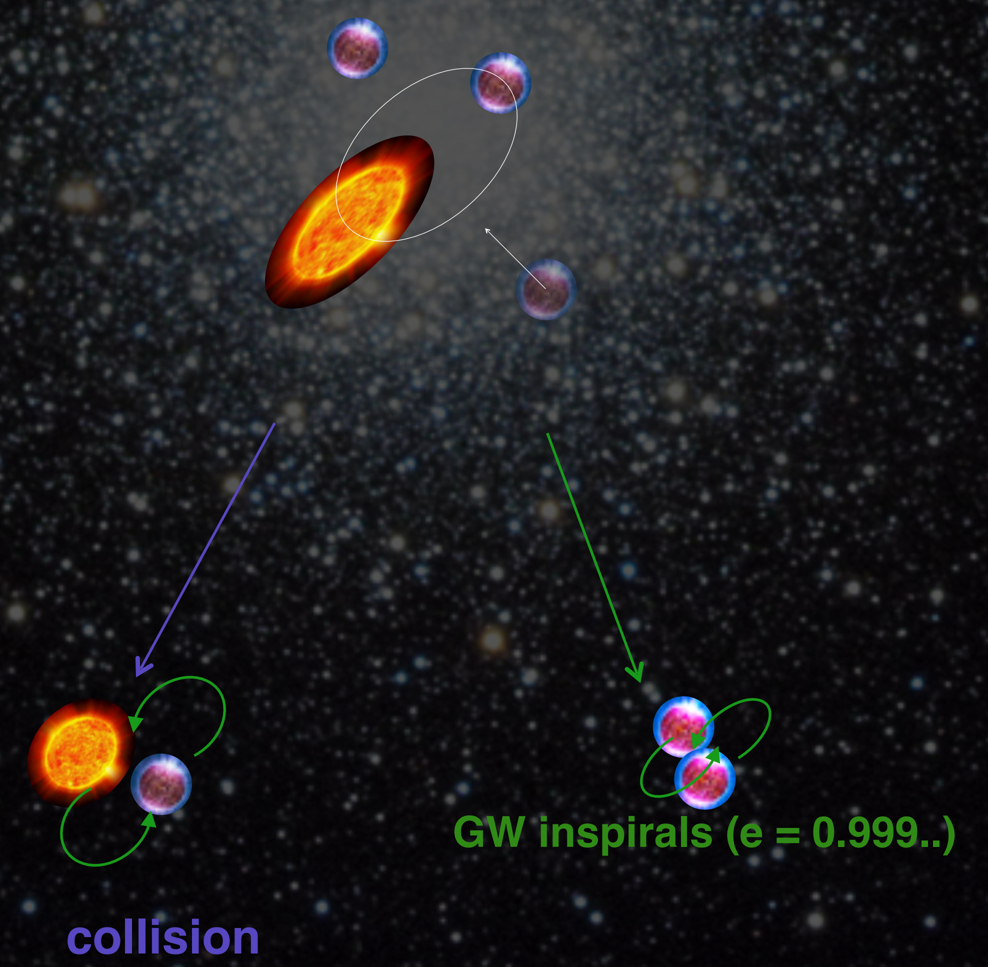


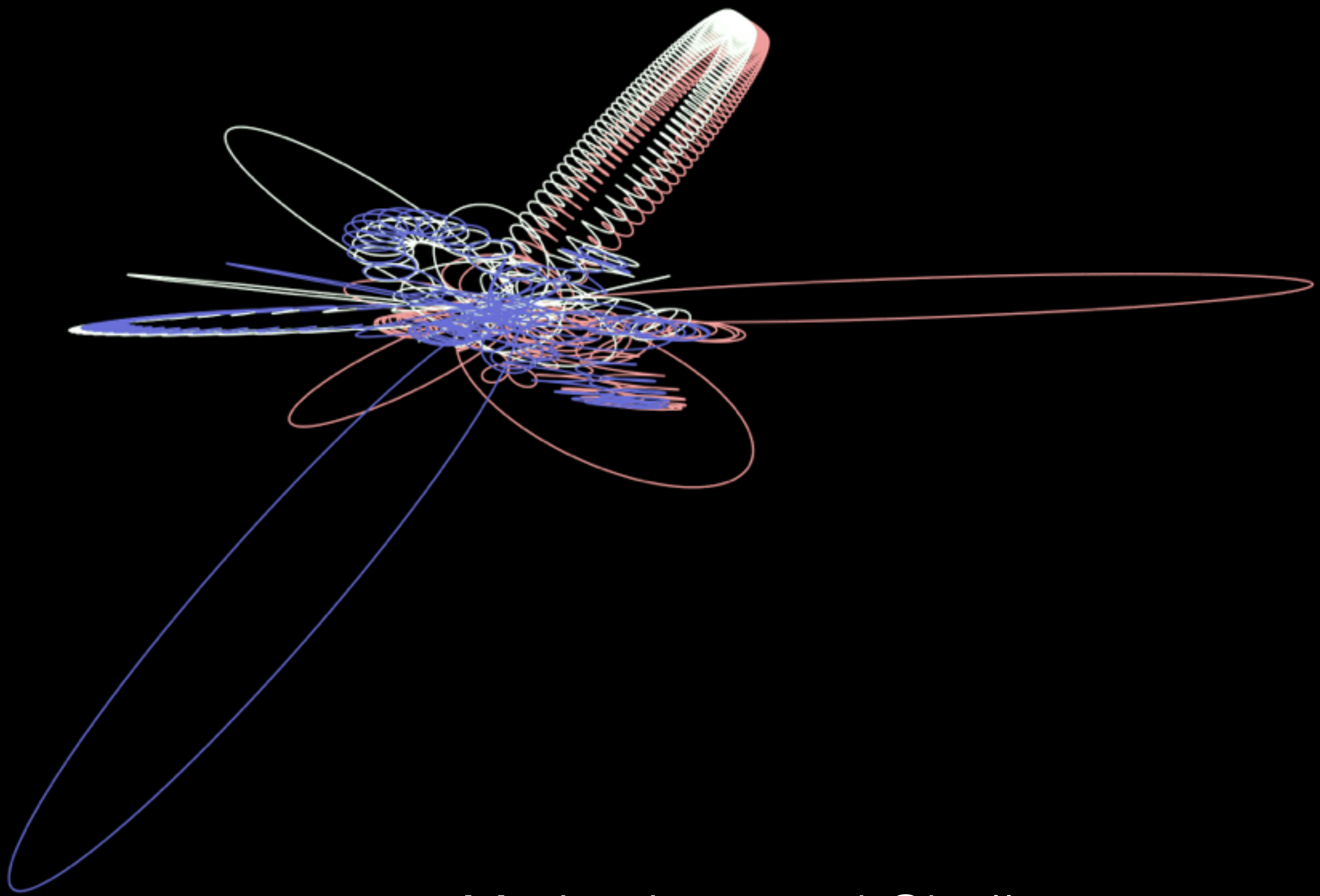
Interaction Channels:

Include: **GR**

Include: **Tides**

Binary-Single



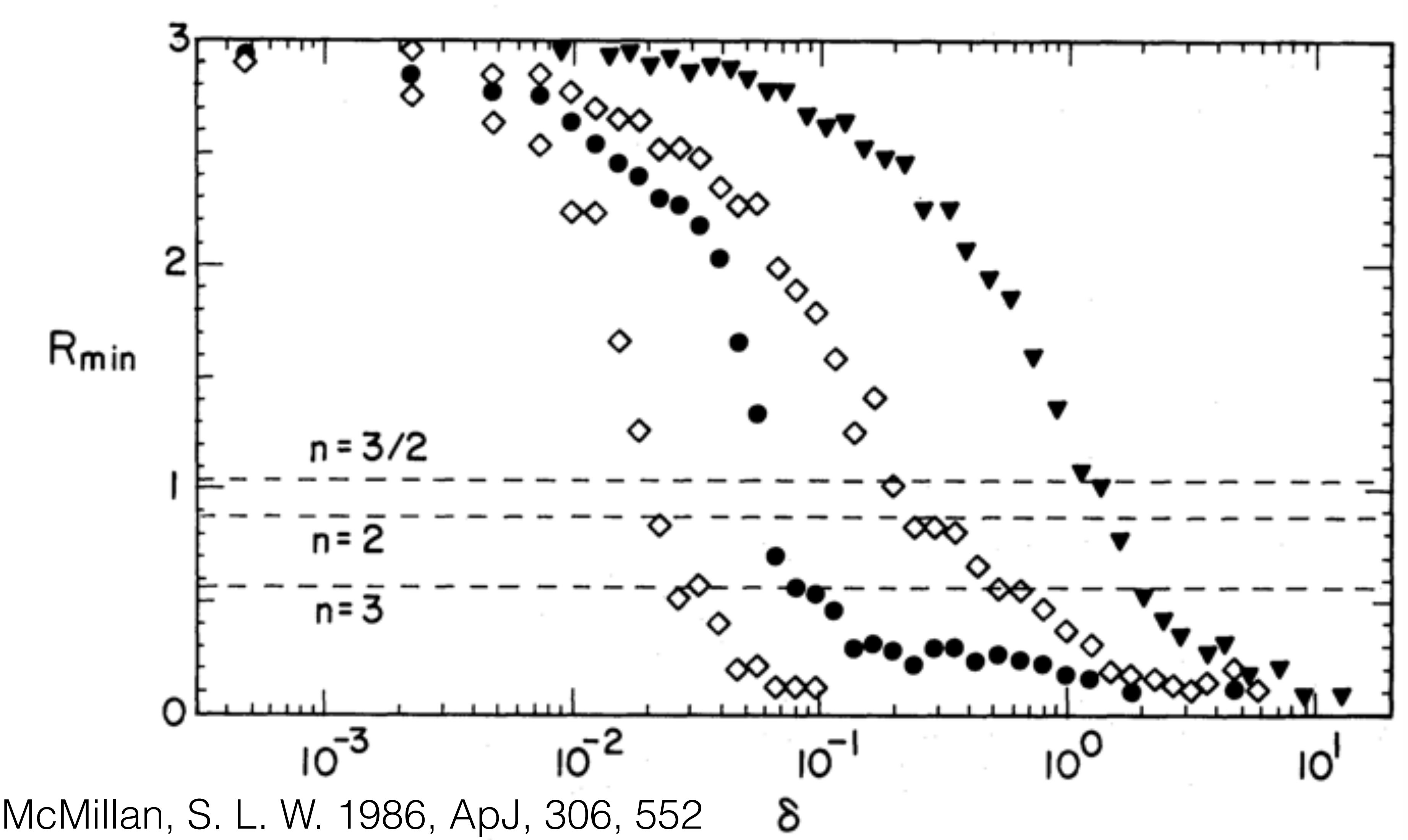


Motivations and Challenges

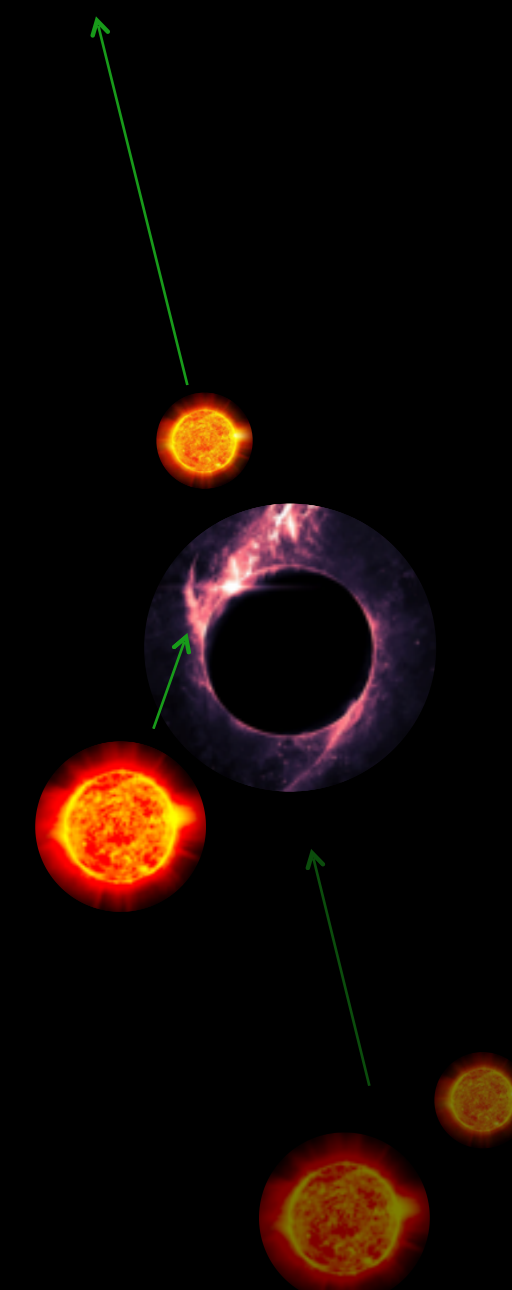
Motivation:

Globular Cluster Dynamics

- What is the proper size of a star when tides and GR are included?
- Exchanges or Tidally Induced Collisions?
- Reduced Energy Kicks?
- Hyper velocity stars through Hills Mechanism (GAIA)

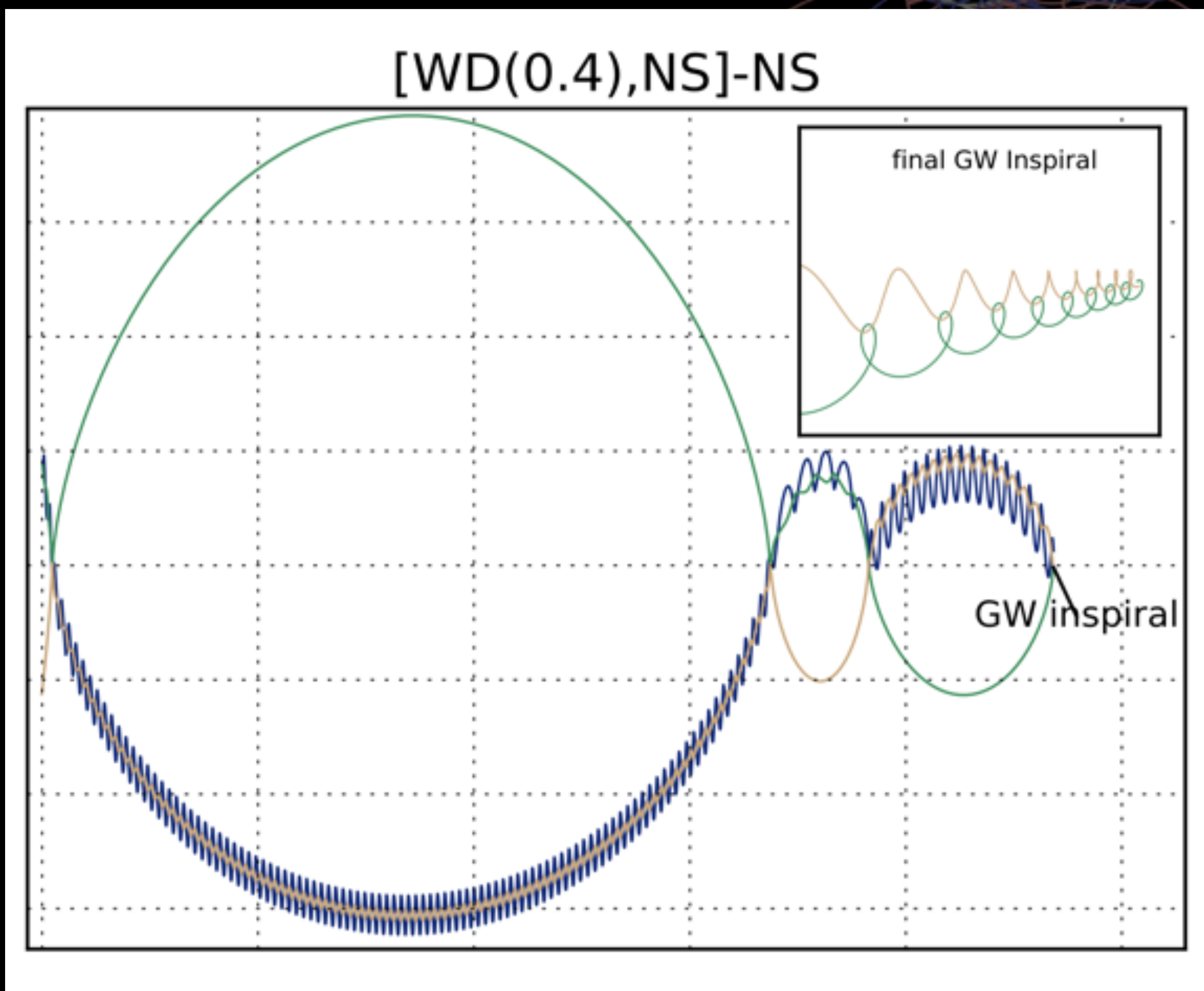


McMillan, S. L. W. 1986, ApJ, 306, 552

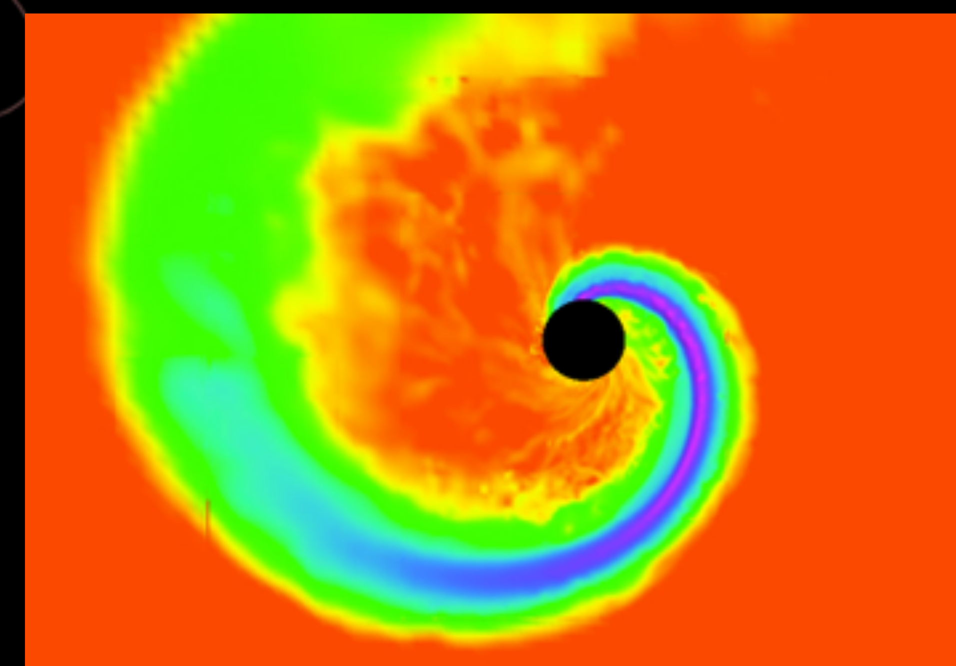


Motivation: Formation of Compact Binaries

- Increased dynamical formation of compact object mergers?
- Hardening of binary BHs or disruptions?
- Formation of sGRB NS-NS binaries.



Full GR:



Pretorius/East

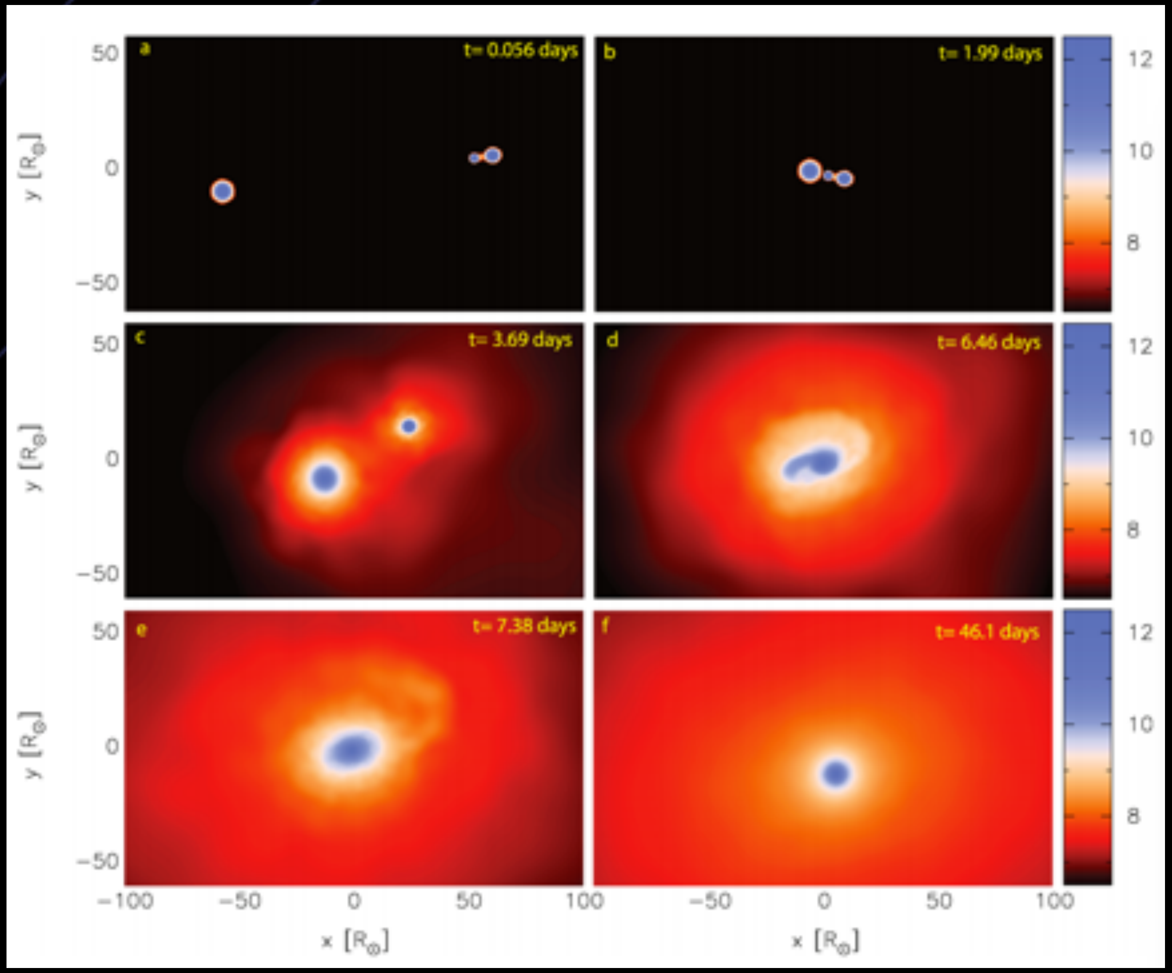
Dominates the high eccentric NS-NS GW inspirals

Challenges:

- Extremely difficult to simulate due to different timescales.
- What tidal model should be used? Dissipation, mode couplings?
- No analytical guidance related to tides in 3-body interactions.

Example:

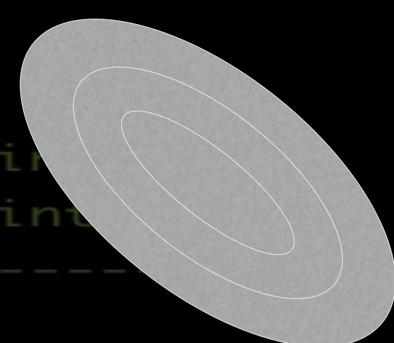
Gaburov, E., Lombardi, Jr., J. C., & Portegies Zwart, S. 2010, MNRAS, 402, 105



N-body Code:

Equations of motion:

- Write out total energy of the system (internal, external) assuming the stars a self-similar ellipses and then apply Euler-Lagrange equations.



$$L_{\mathrm{I}} = T_{\mathrm{I}} - U - \Omega$$

$$L = L_{\mathrm{I}} + L_{\mathrm{C}}$$

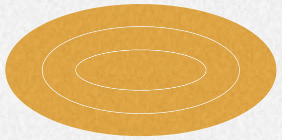
My version:

- Fast Fortran version + MPI MC.
- Hundreds of chaotic orbits in 5-10 sec.
- Single and statistical studies.
- Can never be done with full hydro!!

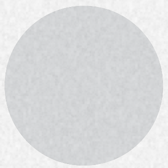
GR model

- PN expansion (v/c)
- 1PN ,2PN ,2.5PN(GWs) order
- Added as modified acc terms.

Affine Model:



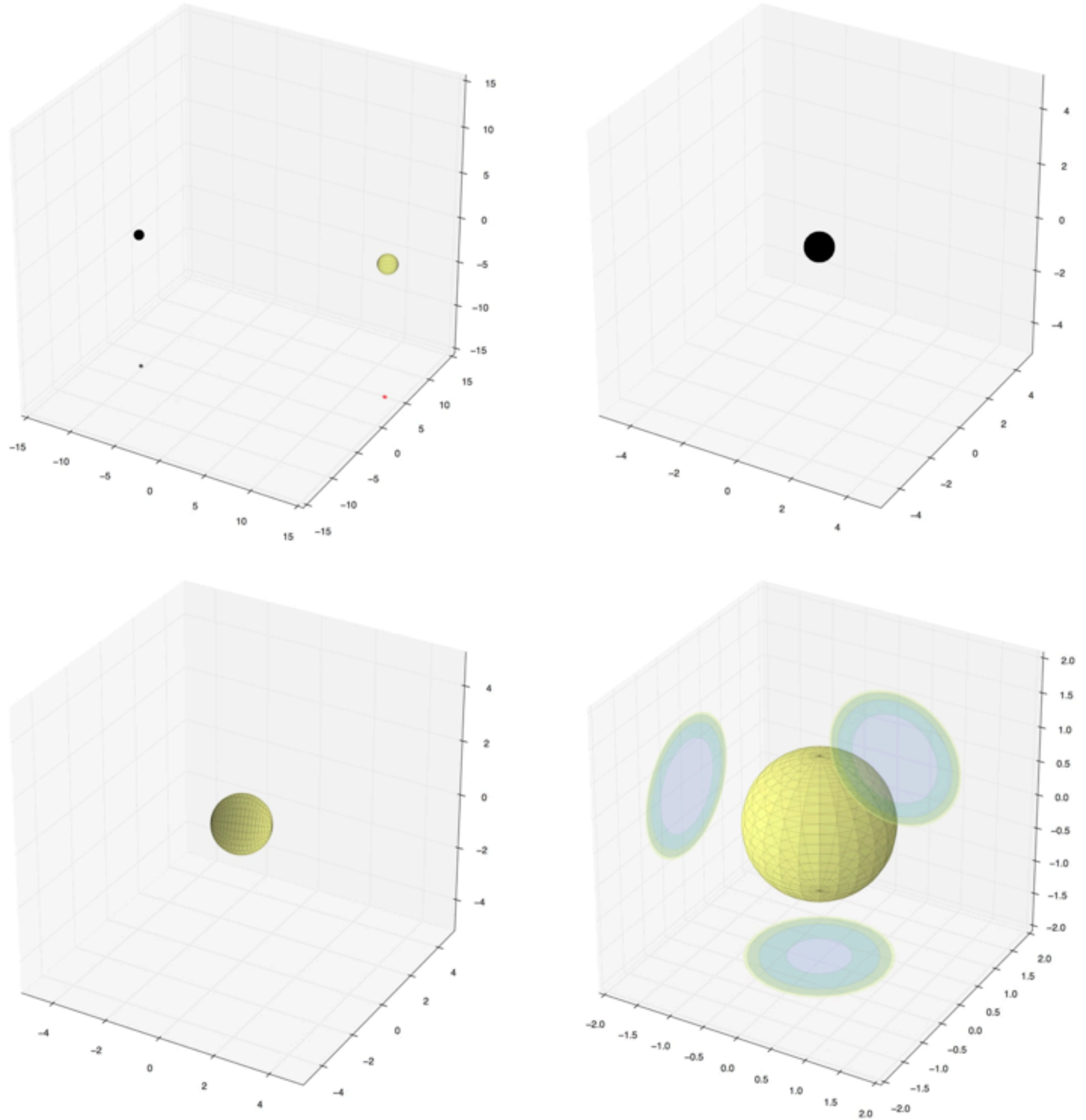
$$r_i = q_{ia}\hat{r}_a$$
$$\dot{r}_i = \dot{q}_{ia}\hat{r}_a$$



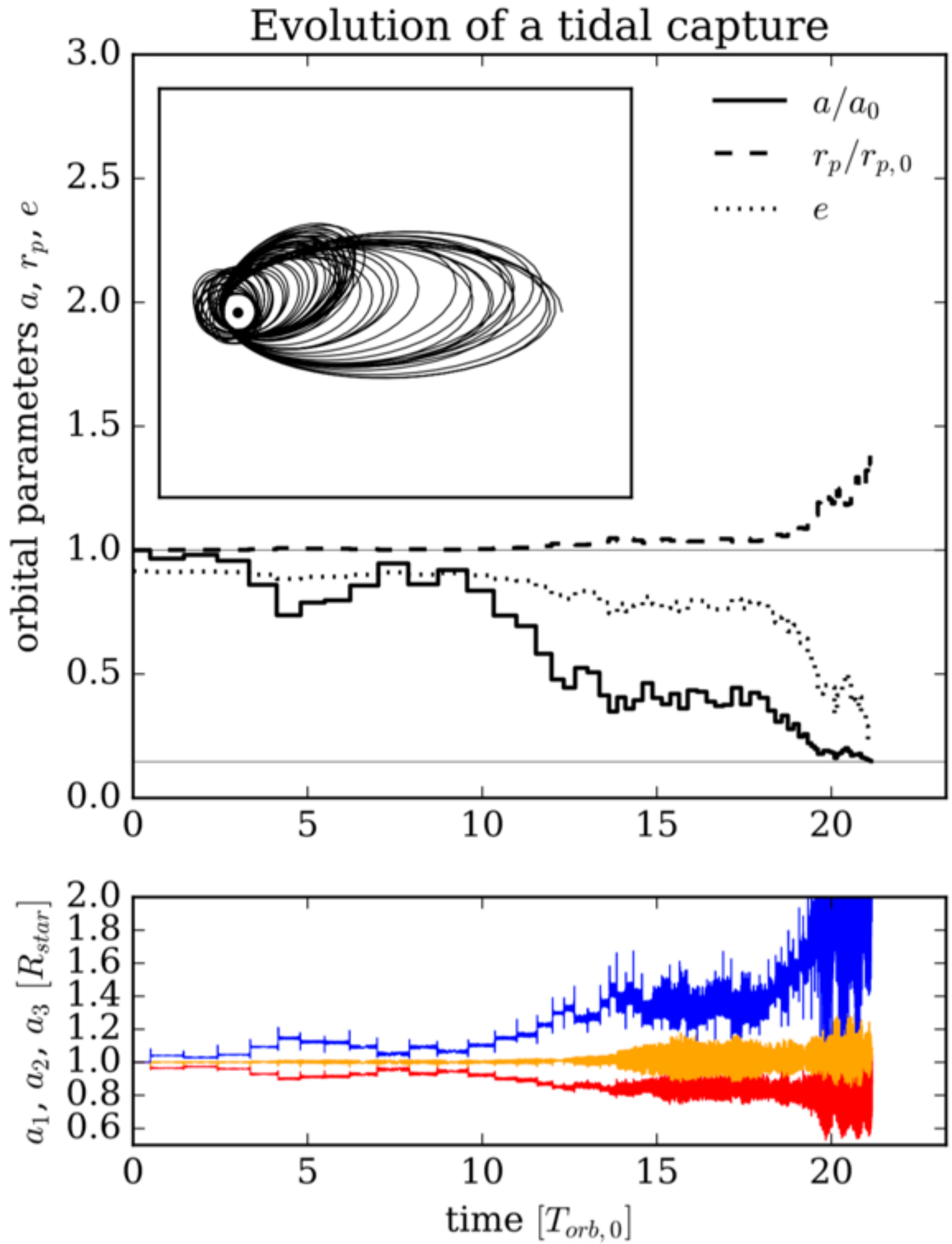
- Self-similar ellipsoids
- Allow non-linear variations
- Polytropic stars
- Fully dynamical
- Easy to add viscosity and GR.
- effectively l=2 (see PT).

2-body studies by:

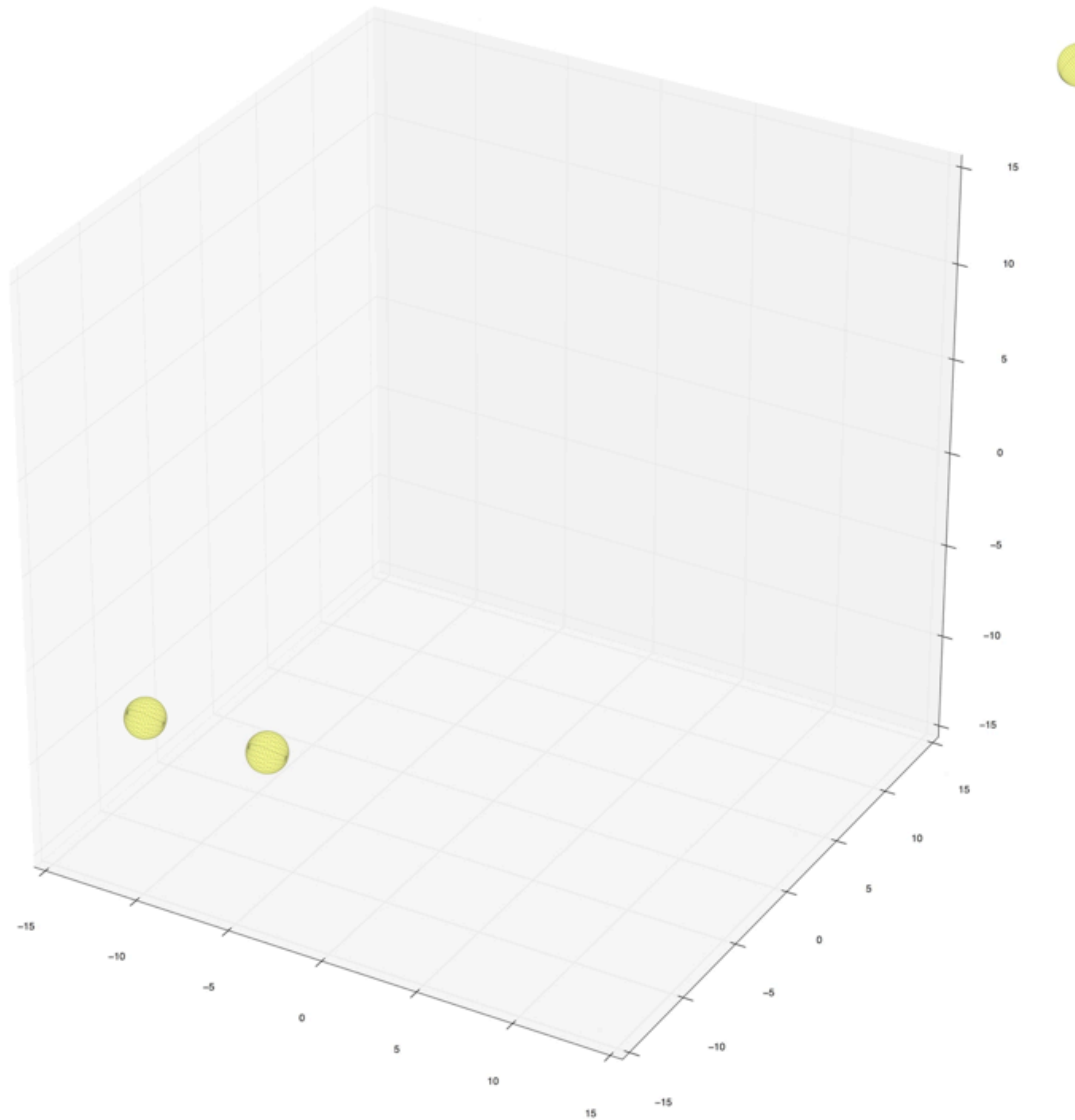
- Carter, Luminet (1985)
- Lai, Rasio, Shapiro (1-4)
- Kochanek (91)



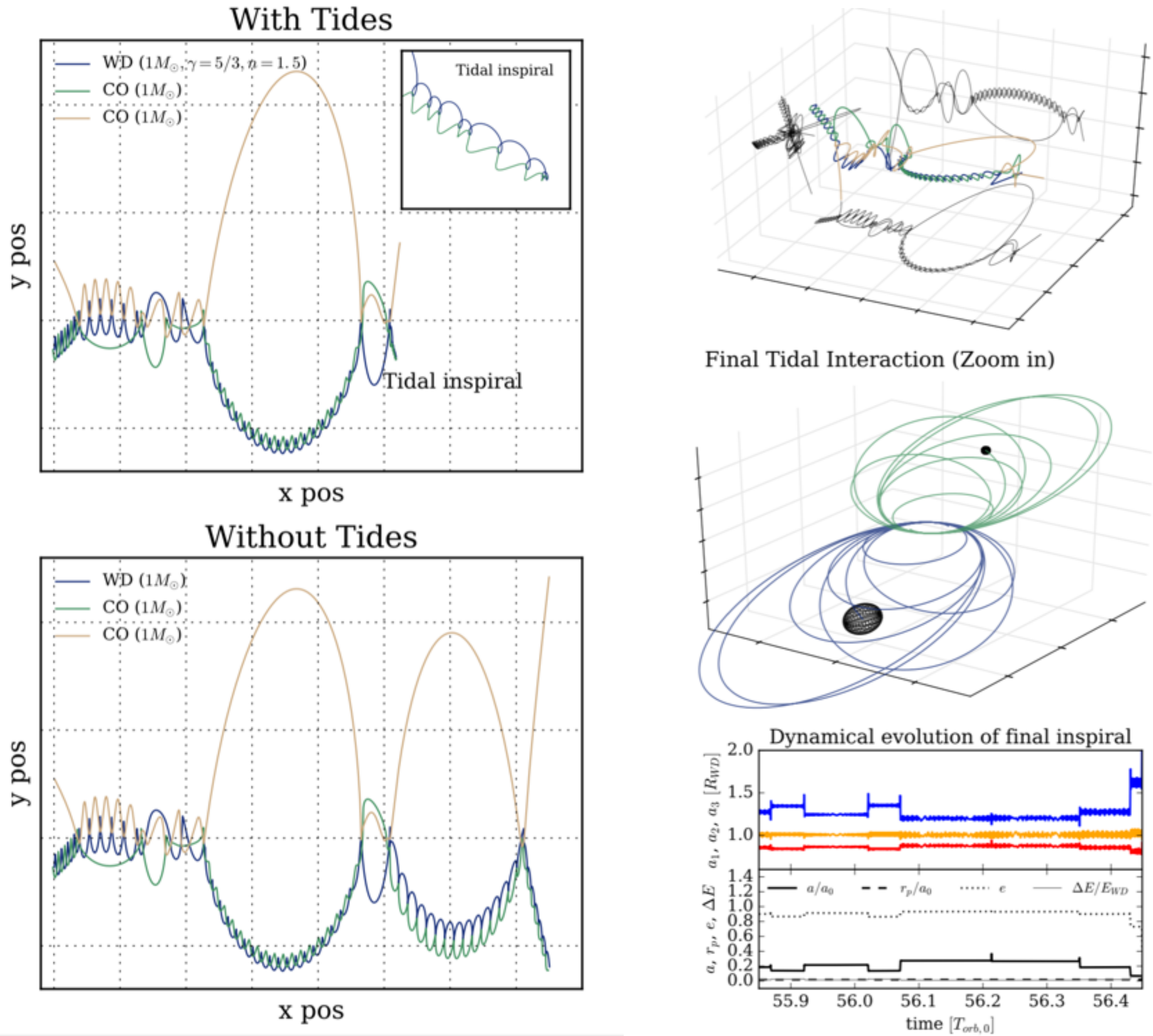
15 10 5 0 −5 −10 −15
−15 −10 −5 0 5 10 15
15 10 5 0 −5 −10 −15
4 2 0 −2 −4
−4 −2 0 2 4
4 2 0 −2 −4
4 2 0 −2 −4
−4 −2 0 2 4
4 2 0 −2 −4
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0



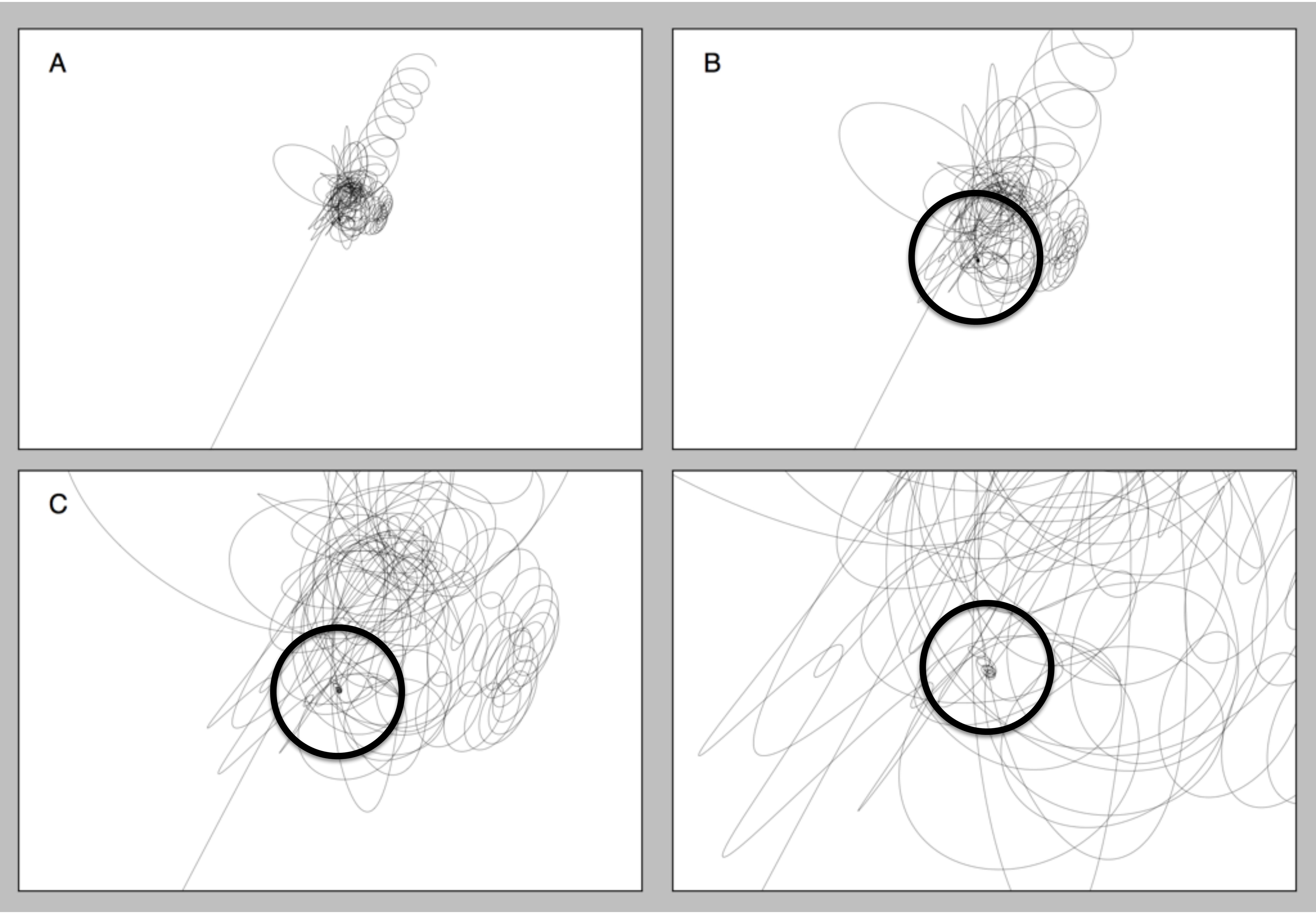
Evolution of a tidal capture
a/a_0
$r_p/r_{p,0}$
e
orbital parameters a, r_p, e
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
a_1, a_2, a_3 $[R_{star}]$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
time $[T_{orb,0}]$



15
10
5
0
−5
−10
−15
−15
−10
−5
0
5
10
15
15
10
5
0
−5
−10
−15

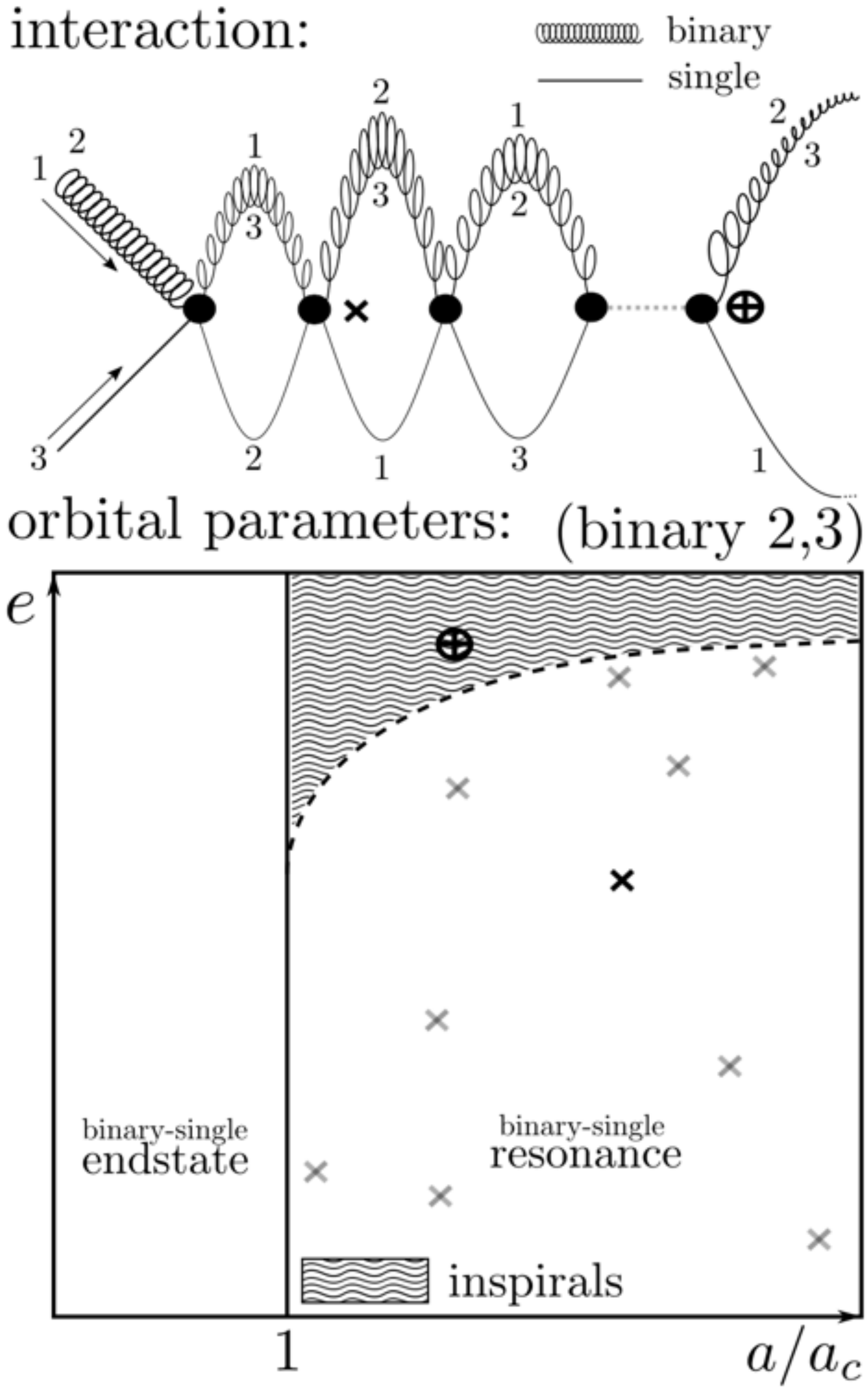


With Tides
WD ($1M_\odot, \gamma = 5/3, n = 1.5$)
CO ($1M_\odot$)
CO ($1M_\odot$)
Tidal inspiral
Tidal inspiral
y pos
x pos
Without Tides
WD ($1M_\odot$)
CO ($1M_\odot$)
CO ($1M_\odot$)
y pos
x pos
Final Tidal Interaction (Zoom in)
Dynamical evolution of final inspiral
a_1, a_2, a_3 $[R_{WD}]$
2.0
1.5
1.0
a, r_p, e, ΔE
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
a/a_0
r_p/a_0
e
$\Delta E/E_{WD}$
55.9
56.0
56.1
56.2
56.3
56.4
time $[T_{orb,0}]$


A
B
C

Formation of GW inspirals and Tidal Captures - Analytical Scaling Solutions

How does the inspiral rate depends
on the initial orbital parameters and the
properties of the interacting
objects (mass, radius, polytropic index etc.)?

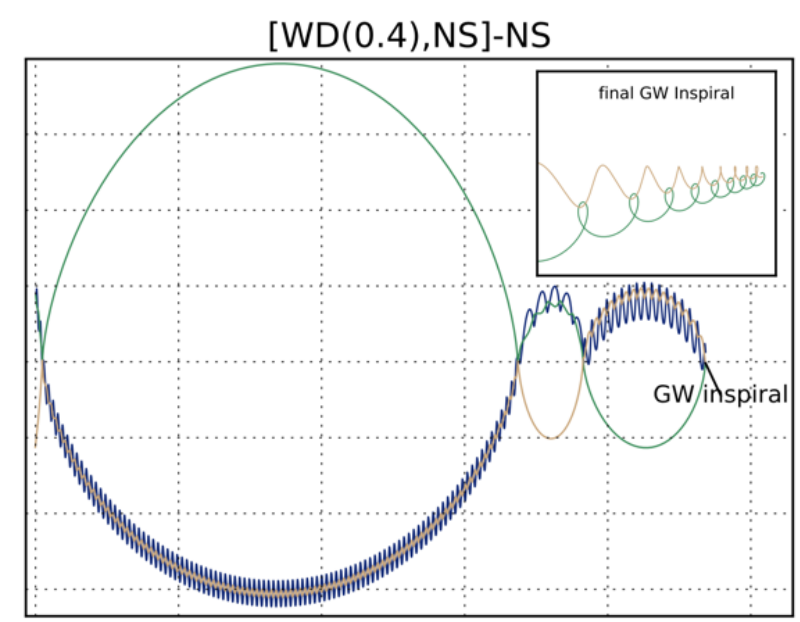


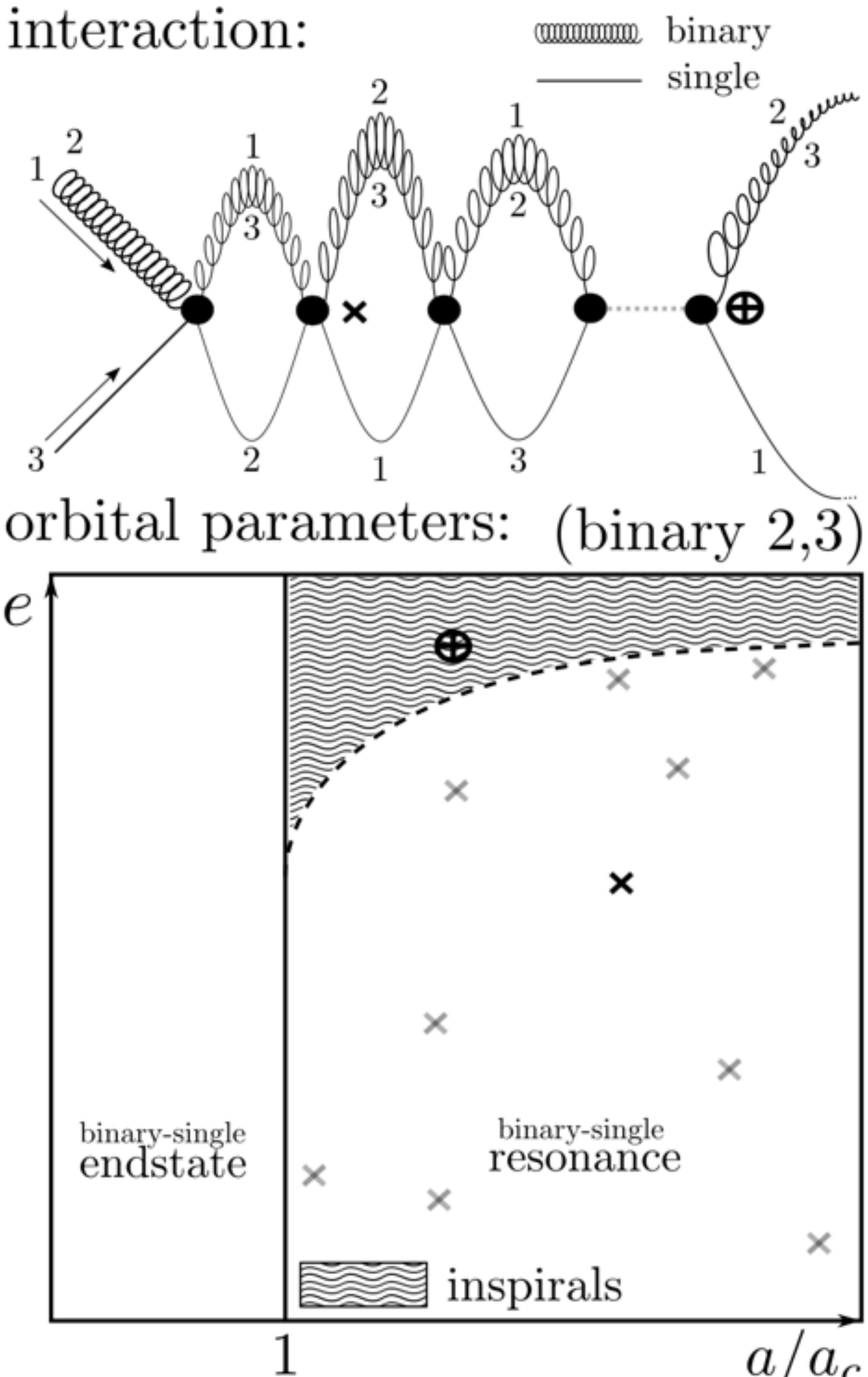
Key ideas:

Find region where inspiral time is less than the isolation time.

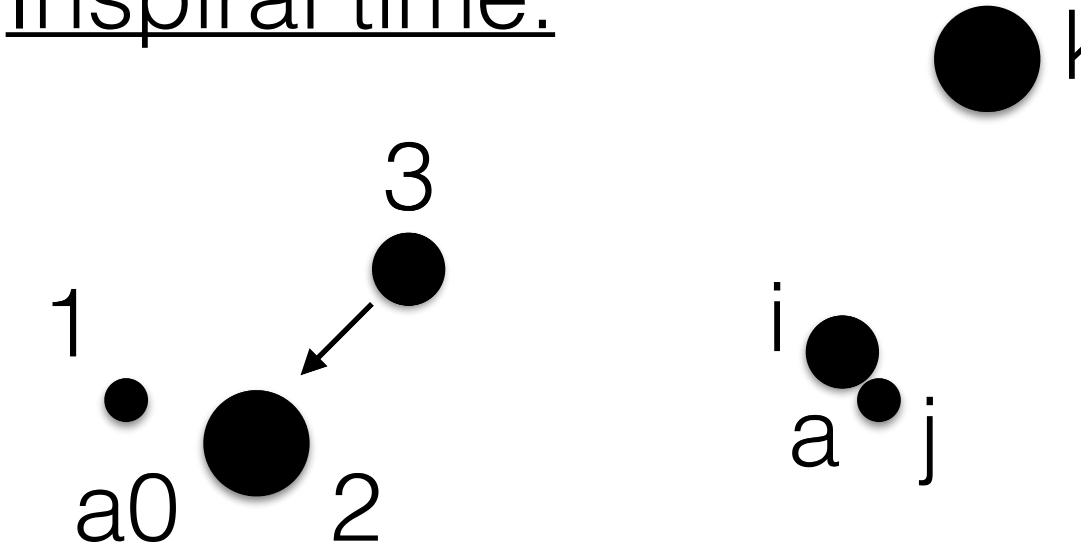
Convert the size of this region to an inspiral probability.

Convert this to a cross section. $\sigma_{I_{ij}} \approx P(I_{ij}|CI) \times \sigma_{CI}$





Inspiral time:



$$\frac{dE}{dt} \approx \frac{\Delta E_p}{T(t)} = \frac{\Delta E_p}{m_{ij}} \frac{\sqrt{2}}{\pi} \mu_{ij}^{-3/2} E(t)^{3/2}$$

$$t_{insp} = 2\pi \sqrt{m_{ij}} \mu_{ij} \frac{\sqrt{a}}{\Delta E_p} = 2T(t_0) \frac{E(t_0)}{\Delta E_p}$$

$$\Delta E_p = \mathscr{E} \frac{M^2}{\mathscr{R}} \left(\frac{\mathscr{R}}{r_p} \right)^{\beta}$$

$$t_{insp} = 2\pi r_p^{\beta} \sqrt{a} \left(\frac{\sqrt{m_{ij}} \mu_{ij} \mathscr{R}^{1-\beta}}{\mathscr{E} M^2} \right)$$

$$\Delta E_p = \mathscr{E} \frac{M^2}{\mathscr{R}} \left(\frac{\mathscr{R}}{r_p}\right)^{\beta}$$

Tides:

$$\Delta E_{tid} \approx \frac{G m_j^2}{R_i} \left(\frac{R_i}{r_p}\right)^6 T_2(\eta)$$

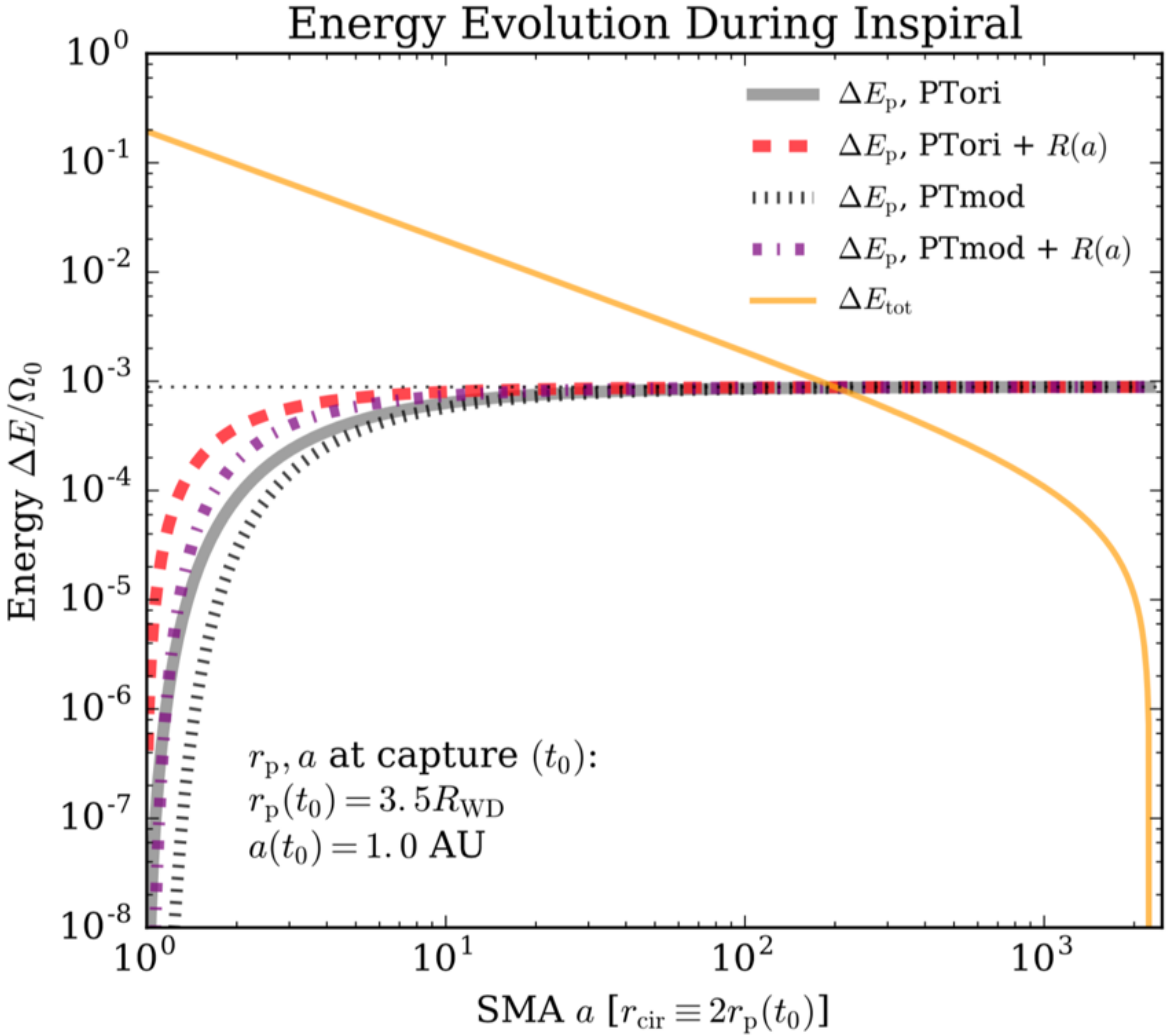
$$T_2(\eta) \approx A \eta^{-\alpha}$$

$$\mathscr{E} = A,\ \mathscr{R} = R_i,\ M = m_j \left(\frac{m_j}{\mu_{ij}}\right)^{\alpha/4},\ \beta = 6 + \frac{3\alpha}{2}$$

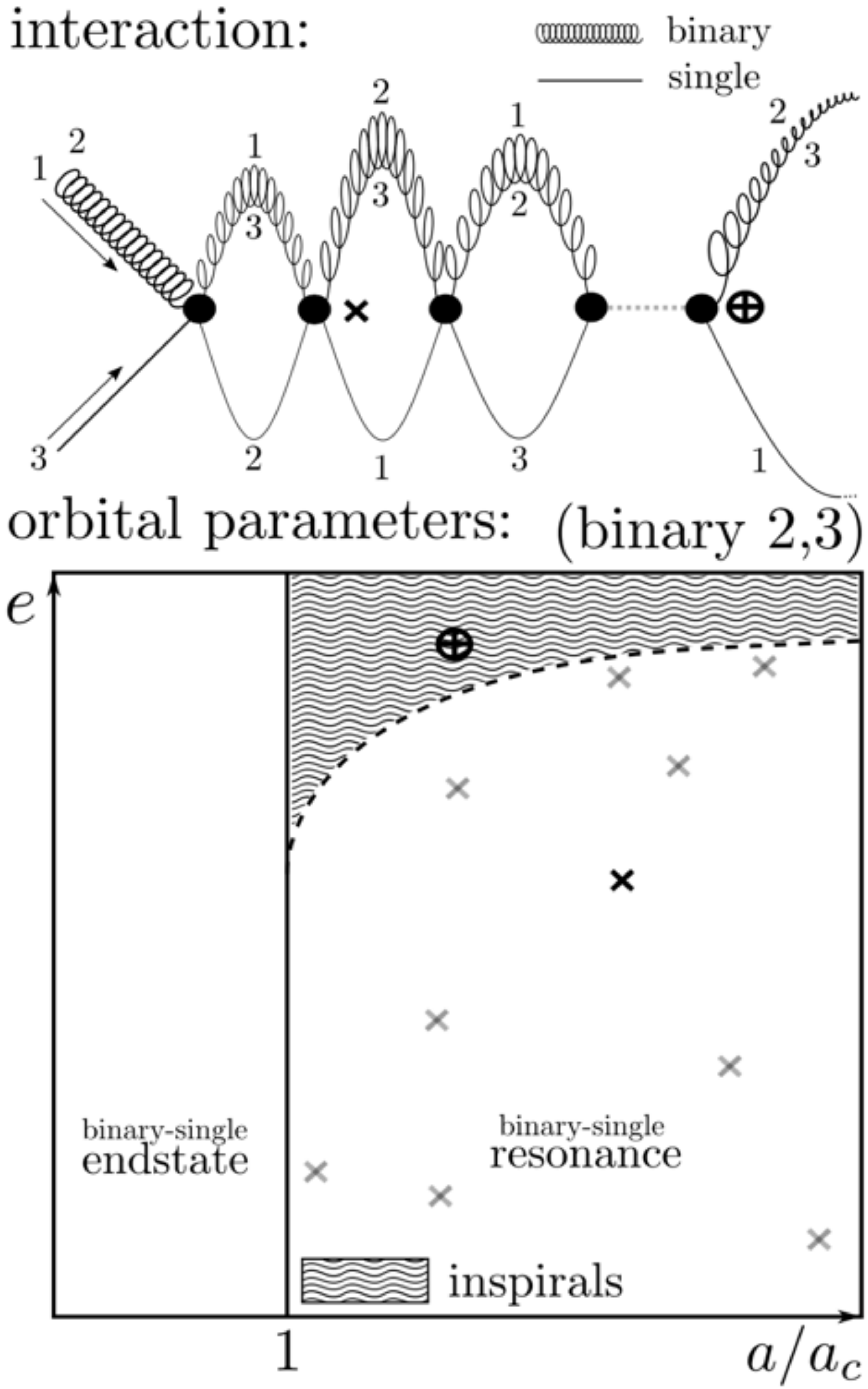
GW radiation

$$\Delta E_{\mathrm{GW}} \approx \frac{85\pi}{12\sqrt{2}} \frac{G^{7/2}}{c^5} \frac{m_i^2 m_j^2 m_{ij}^{1/2}}{r_p^{7/2}}$$

$$\mathscr{E} = \frac{85\pi}{96},\ M = \mu_{ij},\ \mathscr{R} = \frac{2 G m_{ij}}{c^2},\ \beta = \frac{7}{2}$$



Energy Evolution During Inspiral
ΔE_p, PTori
ΔE_p, PTori + R(a)
ΔE_p, PTmod
ΔE_p, PTmod + R(a)
ΔE_tot
Energy ΔE/Ω_0
SMA a [r_cir ≡ 2r_p(t_0)]
r_p, a at capture (t_0):
r_p(t_0) = 3.5R_WD
a(t_0) = 1.0 AU



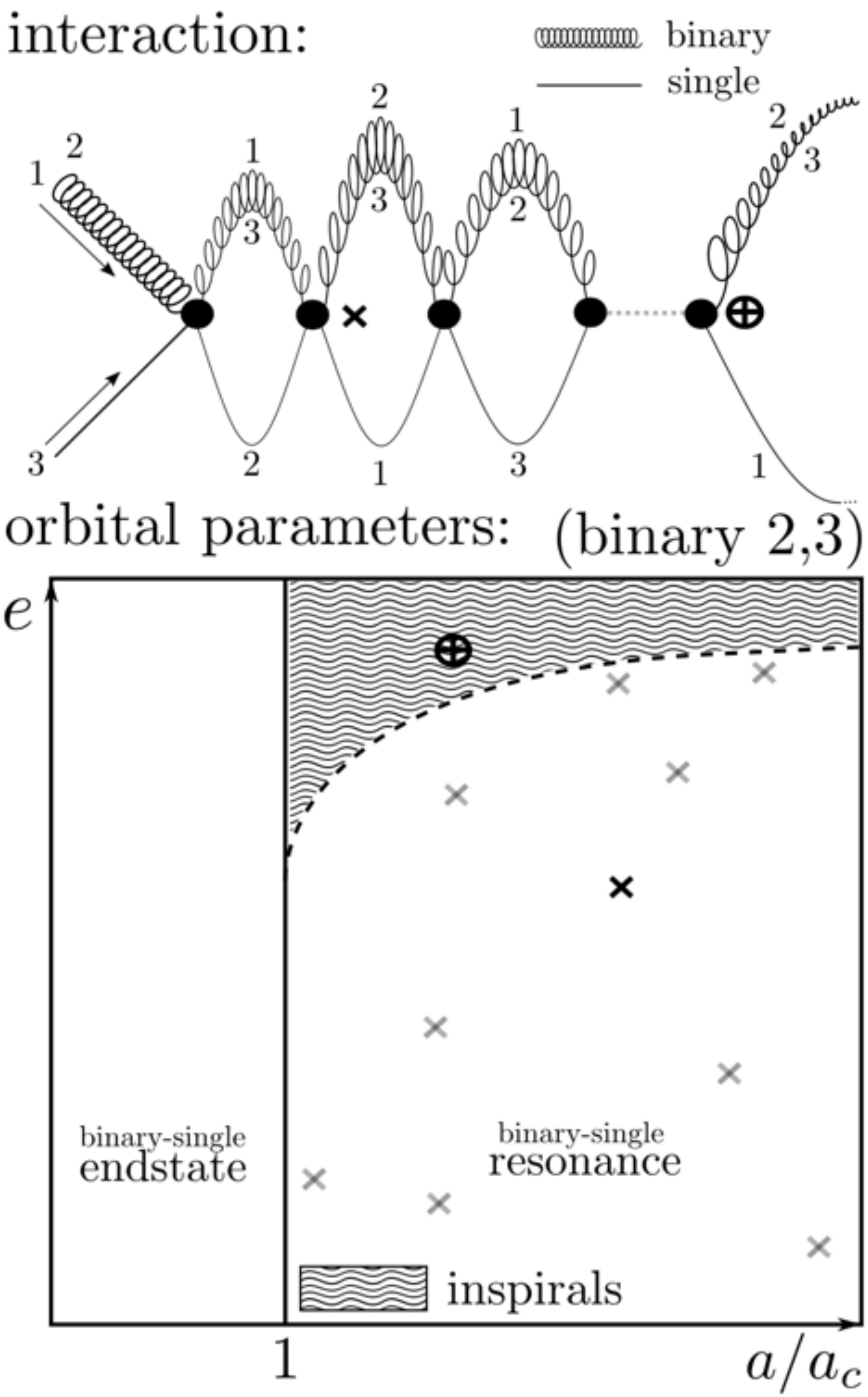
Isolation time:

$$t_{iso} = 2\pi \sqrt{\frac{a_{bs}^3}{m_{bs}}}$$

$$\frac{m_1 m_2}{2a_0} = \frac{m_i m_j}{2a} + \frac{m_{ij} m_k}{2a_{bs}}$$

$$a' \equiv \frac{a}{a_c},\ a_c \equiv a_0 \left(\frac{m_i m_j}{m_1 m_2}\right)$$

$$t_{iso} = 2\pi \frac{a_0^{3/2}}{\sqrt{m_{bs}}} \left(\frac{m_{ij} m_k}{m_1 m_2}\right)^{3/2} \left(\frac{a'}{a'-1}\right)^{3/2}$$



Inspiral boundary:

$$\epsilon_{insp} = \mathscr{E}^{1/\beta} \mathscr{M} \left(a_0/\mathscr{R}\right)^{(1/\beta-1)} \mathscr{G}(a',\beta)$$

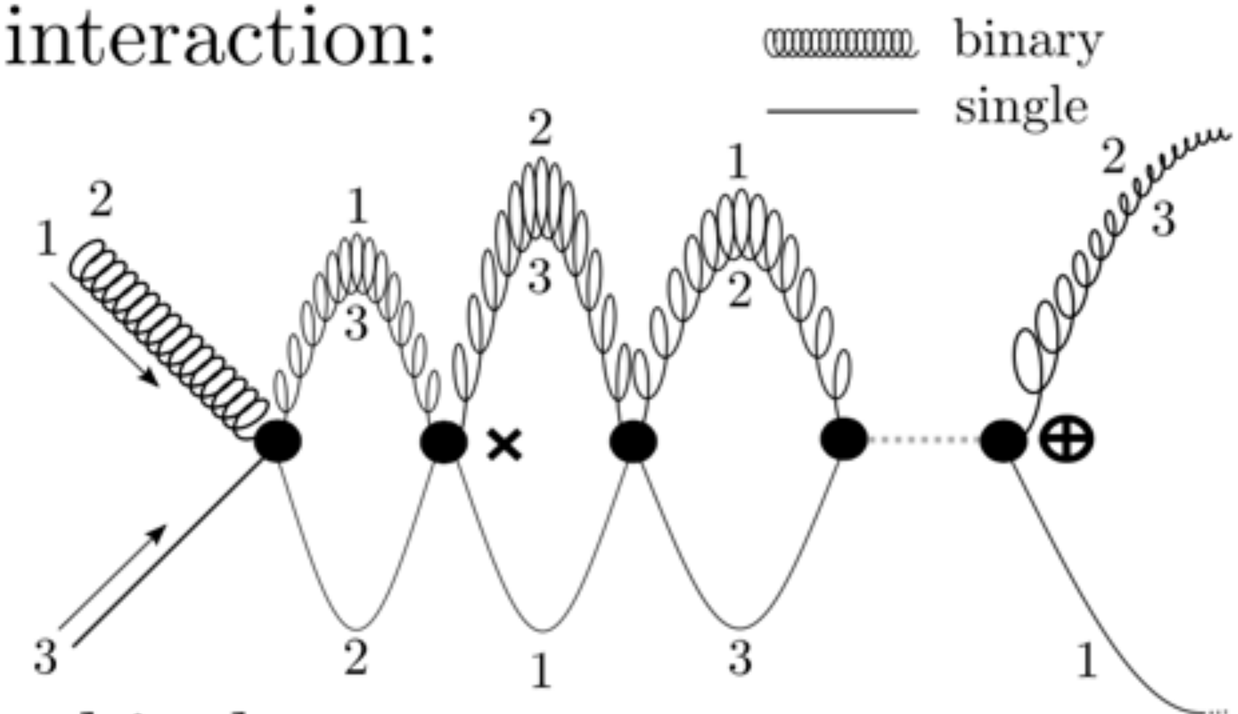
$$\epsilon_{insp} \equiv (1 - e_{insp})$$

$$\mathscr{G}(a',\beta) = a'^{(1/\beta-1)} \left(a'-1\right)^{-3/(2\beta)}$$

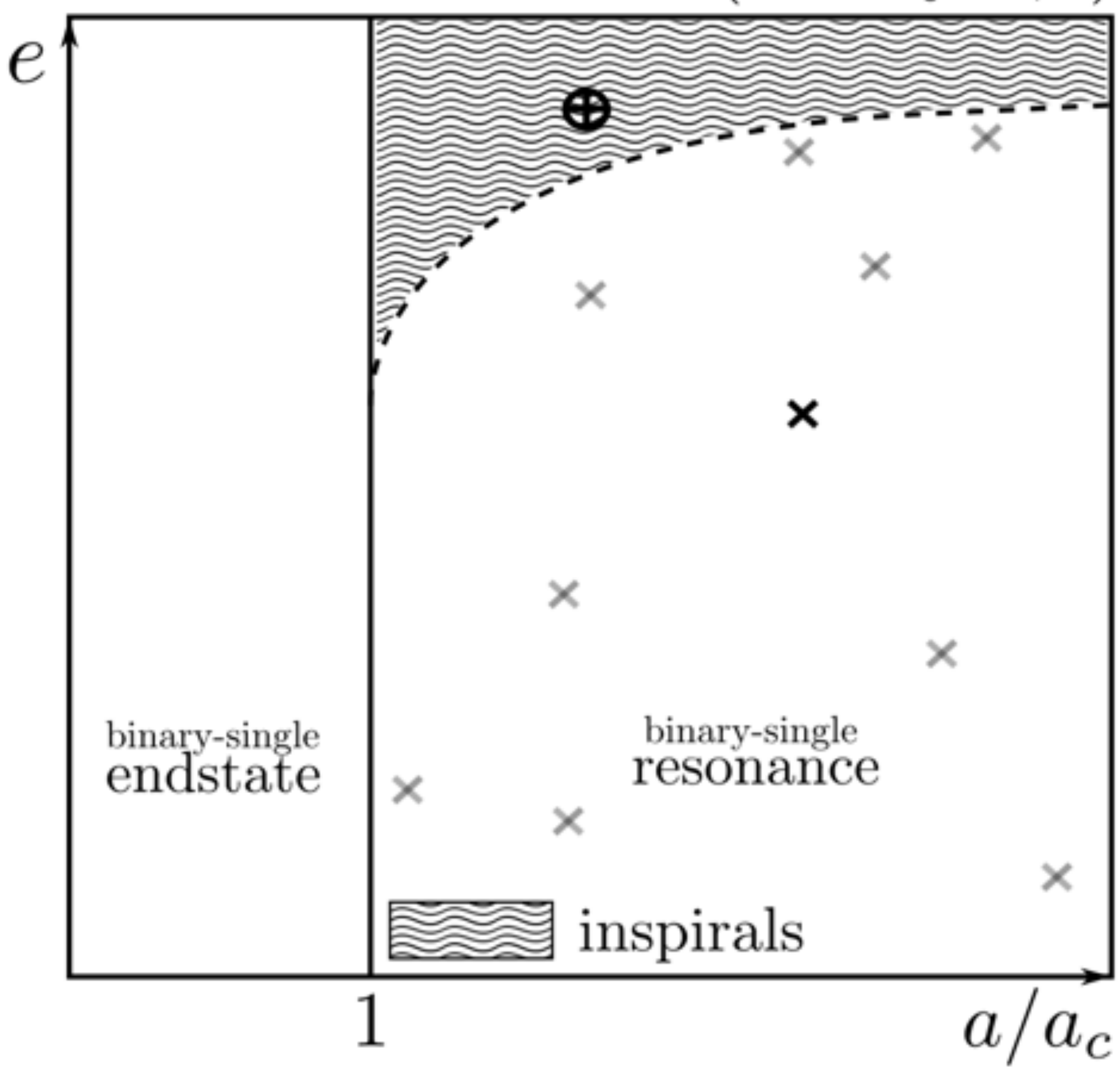
$$\mathscr{M} = \left(\frac{m_1 m_2}{m_i m_j}\right) \left[\left(\frac{M}{m_{bs}}\right)^2 \left(\frac{m_{bs}}{\mu_{ij}}\right)^{\frac{3}{2}} \left(\frac{m_k m_k}{m_1 m_2}\right) \left(\frac{m_{ij}}{m_k}\right)^{\frac{1}{2}}\right]^{1/\beta}$$

Lets consider an example

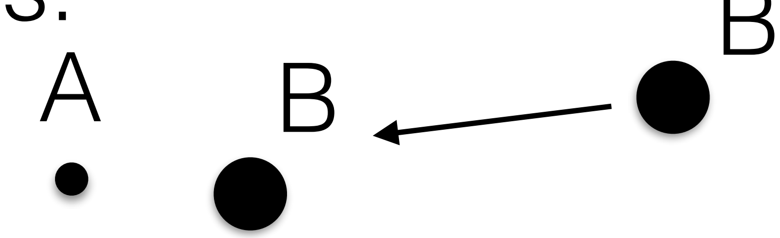
interaction:



orbital parameters: (binary 2,3)



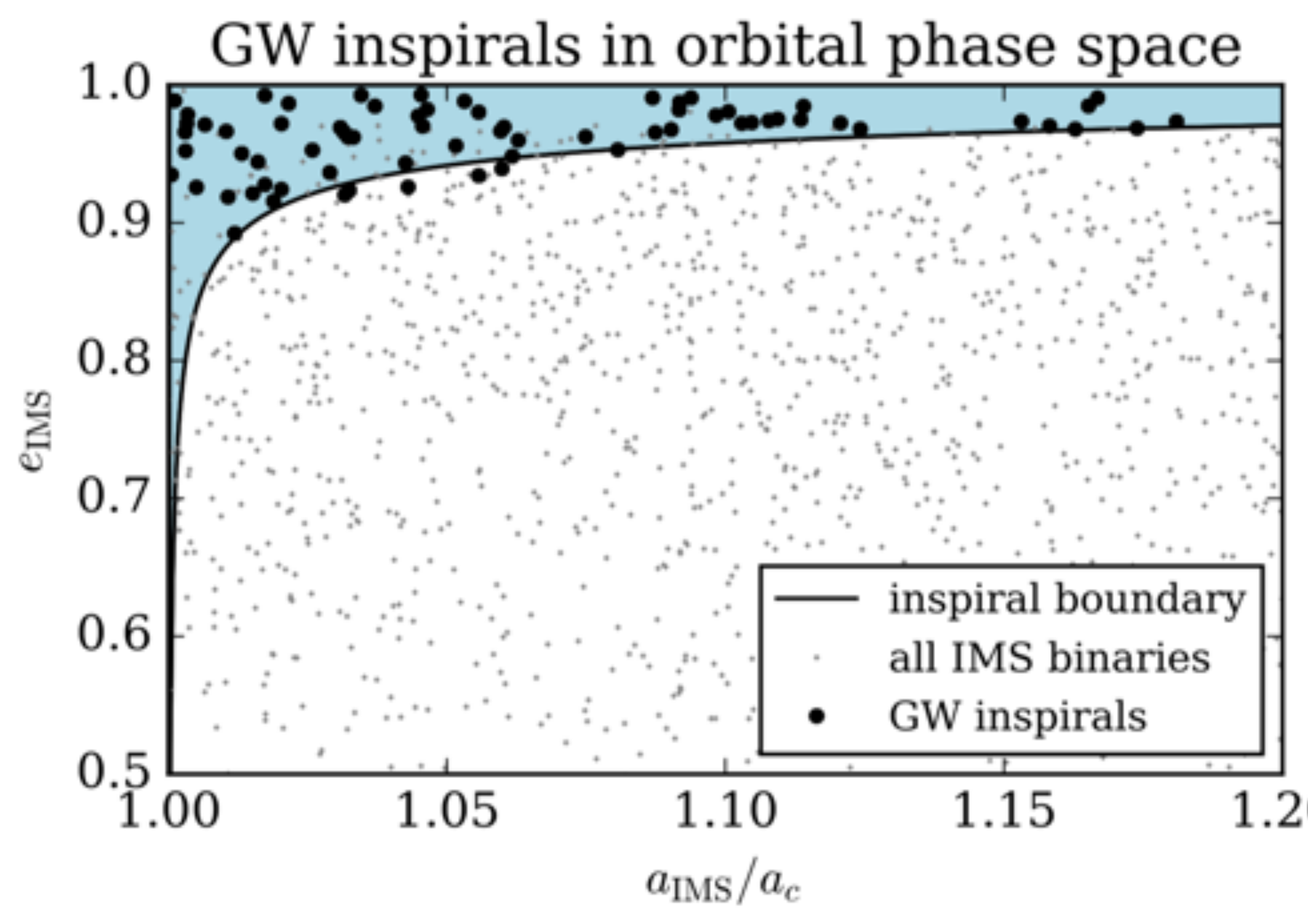
BH inspirals:

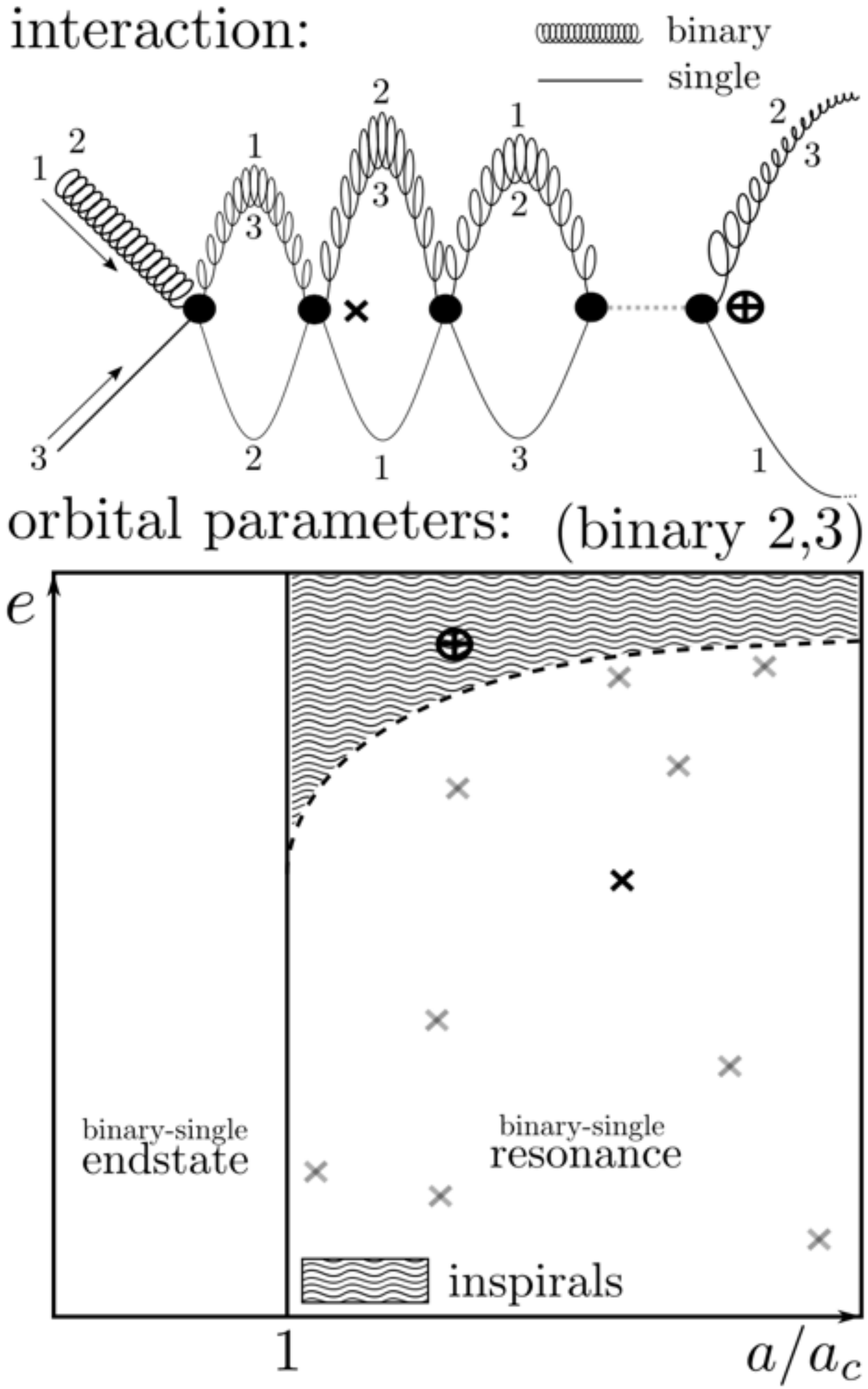


$$\tilde{\epsilon}_{\rm GW}(m, a_0) = C_{\rm GW} \times \frac{G^{5/7} m^{5/7}}{c^{10/7} a_0^{5/7}} \mathscr{G}(a', \beta = 7/2)$$

$$\epsilon_{\rm GW} = \tilde{\epsilon}_{\rm GW}(m_B) \left[q \left(\frac{3q}{2+q} \right)^{1/7} \right]$$

$$C_{\rm GW} = \left(\frac{85\pi}{3\sqrt{3}} \right)^{2/7} \qquad q \equiv \frac{m_A}{m_B}$$





Inspiral boundary:

$$\epsilon_{insp} = \mathscr{E}^{1/\beta} \mathscr{M} \left(a_0/\mathscr{R}\right)^{(1/\beta-1)} \mathscr{G}(a', \beta)$$

$$\epsilon_{insp} \equiv (1 - e_{insp})$$

$$\mathscr{G}(a', \beta) = a'^{(1/\beta-1)} \left(a' - 1\right)^{-3/(2\beta)}$$

$$\mathscr{M} = \left(\frac{m_1 m_2}{m_i m_j}\right) \left[\left(\frac{M}{m_{bs}}\right)^2 \left(\frac{m_{bs}}{\mu_{ij}}\right)^{\frac{3}{2}} \left(\frac{m_k m_k}{m_1 m_2}\right) \left(\frac{m_{ij}}{m_k}\right)^{\frac{1}{2}}\right]^{1/\beta}$$

Convert to cross section

Cross sections:

Inspirals:

$$\sigma_{I_{ij}} \approx \mathscr{D} \left(\frac{a_0}{\mathscr{R}}\right)^{1/\beta} \left[\mathscr{N} \left(\frac{m_3}{\mu_{12}}\right)^{1/3} \left(\frac{2\pi G m_{bs} \mathscr{R}}{v_\infty^2}\right) \left(\frac{m_1 m_2}{m_i m_j}\right) \ln(a'_u)\right]$$

$$\mathscr{D} \equiv \mathscr{E}^{1/\beta} \mathscr{I}' \mathscr{M}'$$

Collisions:

$$\sigma_{\mathcal{R}_{ij}} \approx \mathscr{N} \left(\frac{m_3}{\mu_{12}}\right)^{1/3} \left(\frac{2\pi G m_{bs} \mathcal{R}}{v_\infty^2}\right) \left(\frac{m_1 m_2}{m_i m_j}\right) \ln(a'_u)$$

Inspirals relative to collisions:

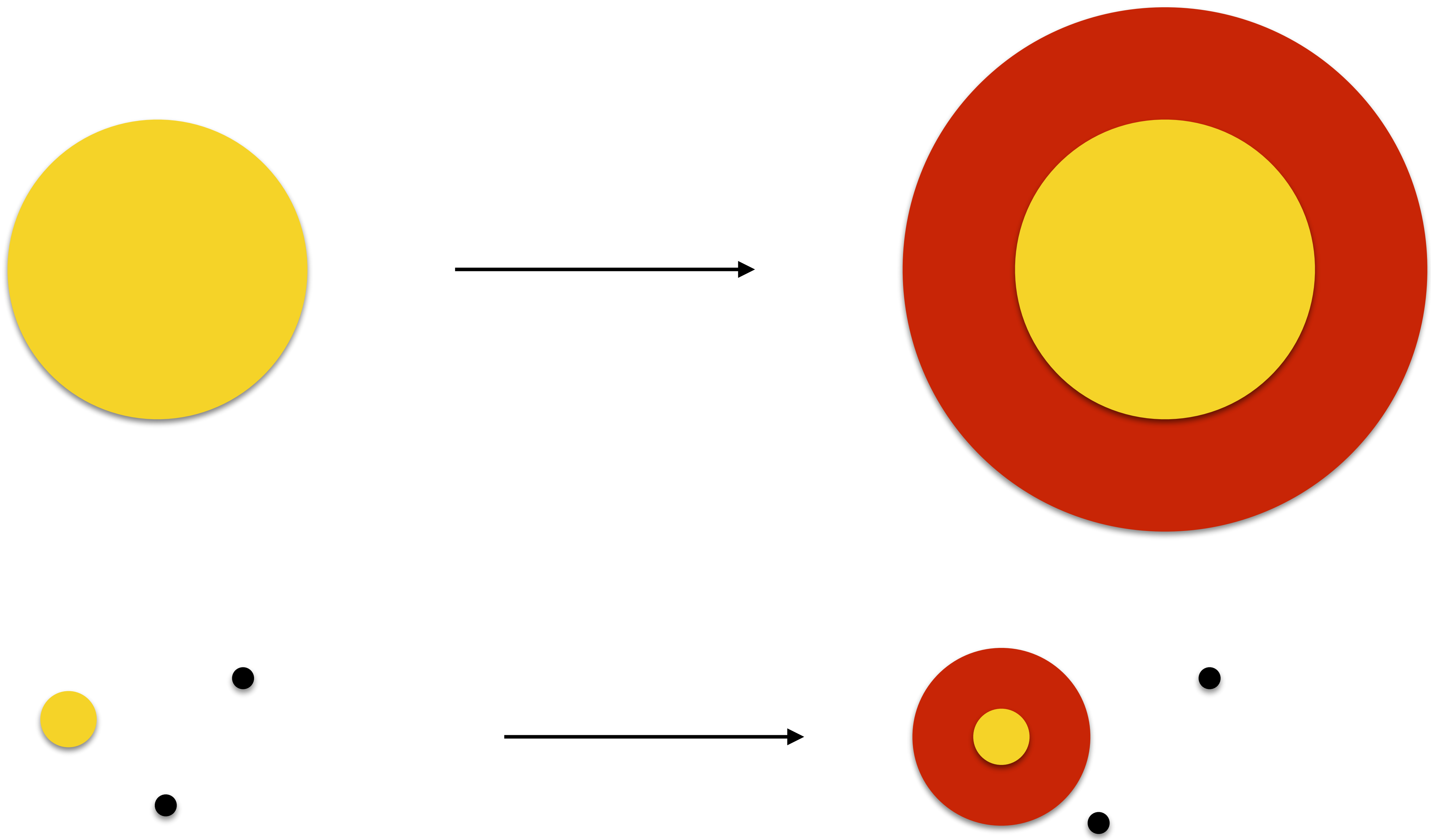
$$\frac{\Gamma_{\mathrm{I_{ij}}}}{\Gamma_{\mathcal{R}_{\mathrm{ij}}}} \approx \frac{A_I - A_{\mathcal{R}}}{A_{\mathcal{R}}} \approx \mathscr{E}^{1/\beta} \mathscr{I}' \mathscr{M}' \mathcal{R}'_{ij} \left(\frac{a_0}{R_i}\right)^{1/\beta} - 1$$

Largest effect for wide binaries and when the objects are small!

Inspirals can greatly dominate over collisions (sticky star approximation)

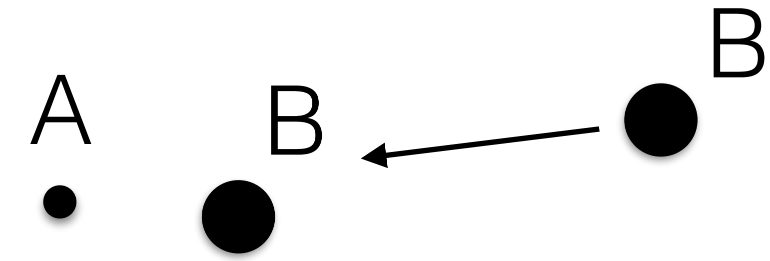
Simple Model

$$\Delta E_{\rm tid} \propto \frac{m^2}{R}\left(\frac{R}{r_{\rm p}}\right)^{\beta} \qquad E_0 \propto \frac{m^2}{a_0} \qquad r_{\rm tid} \propto R\left(\frac{a_0}{R}\right)^{1/\beta}$$



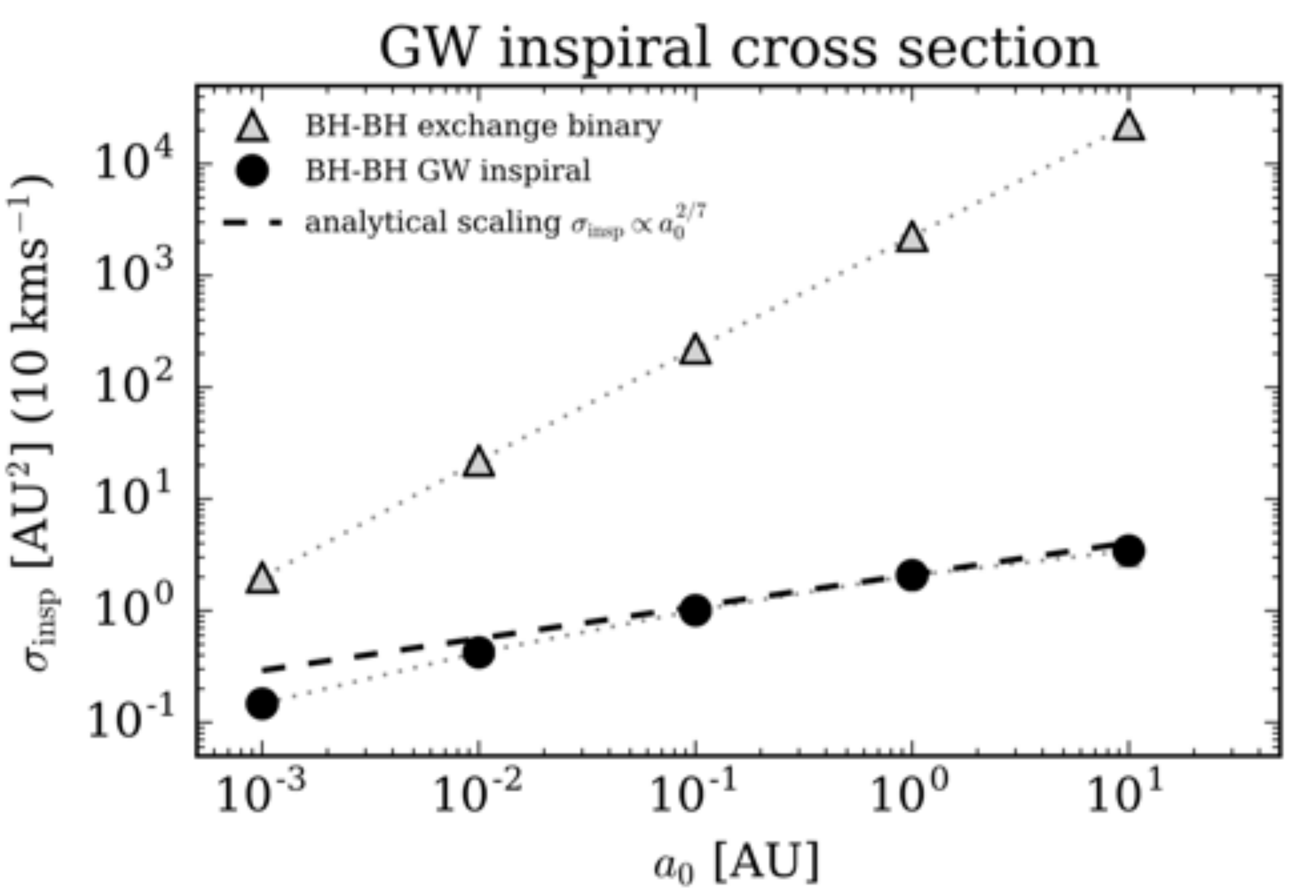
Cross sections: BH-BH example

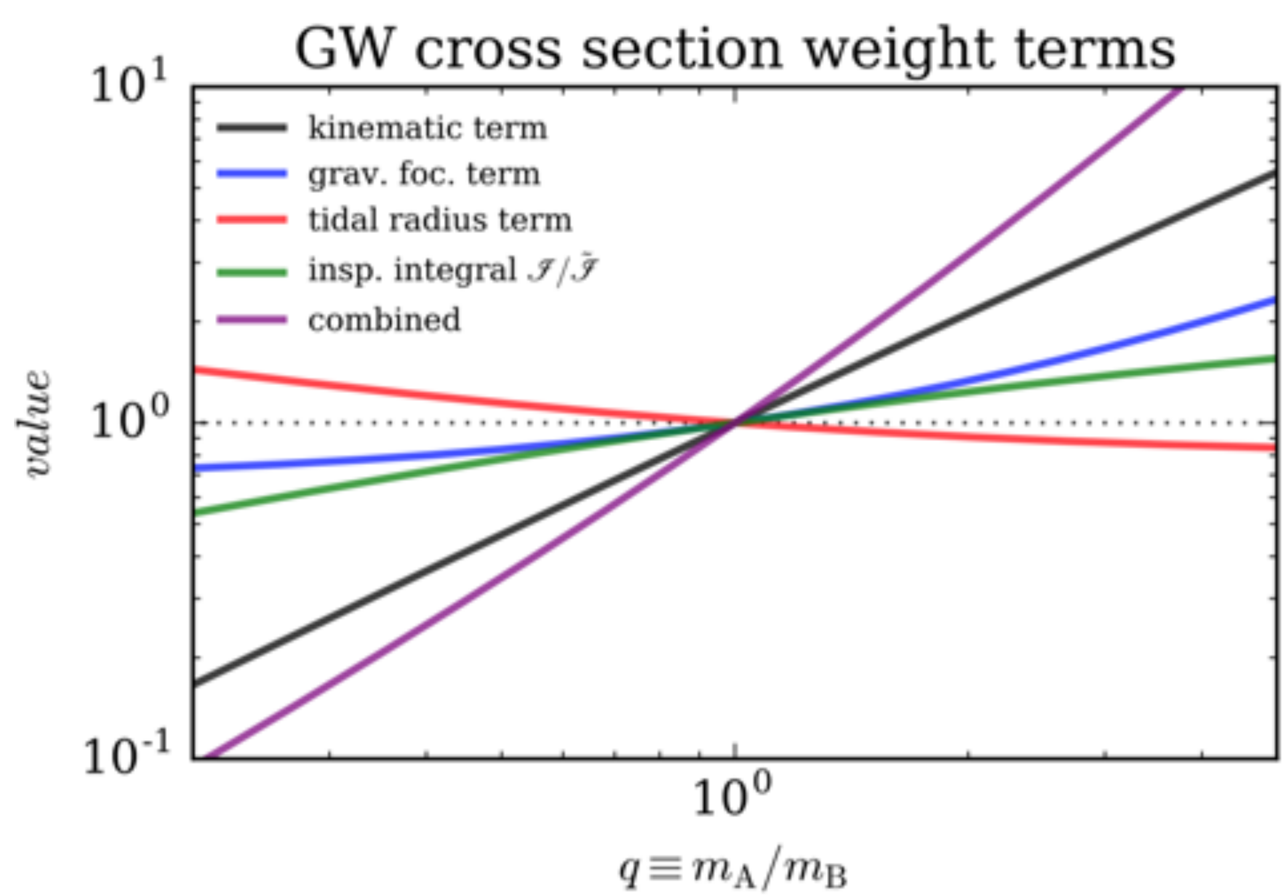
Inspirals:



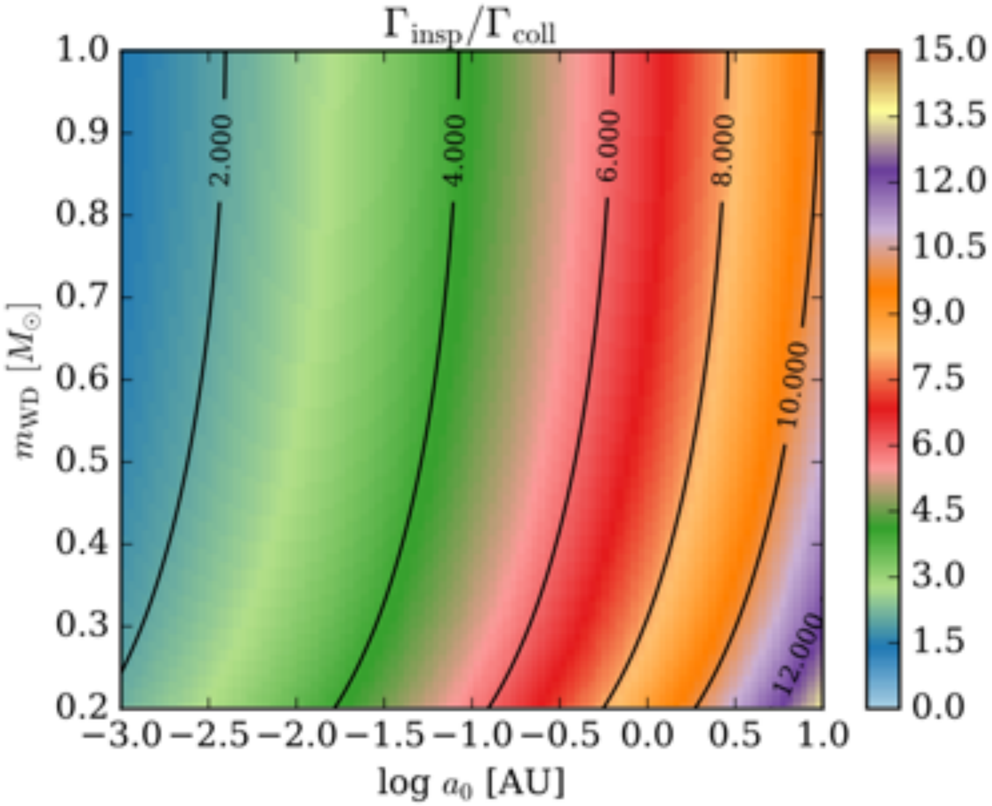
$$\tilde{\sigma}_{\rm GW}(m, a_0, v_\infty) \approx C_{\rm GW}\pi\sqrt[3]{432} \times \tilde{\mathscr{I}}\tilde{\mathscr{N}}\frac{G^{12/7}m^{12/7}a_0^{2/7}}{c^{10/7}v_\infty^2}$$

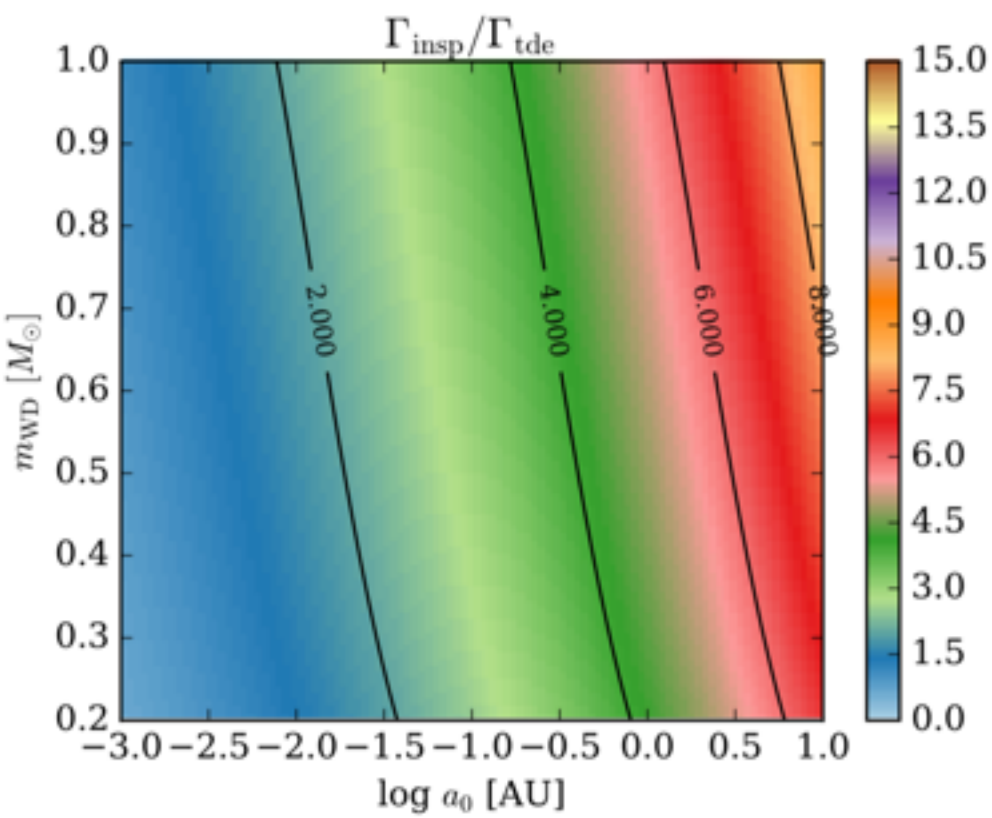
$$\sigma_{\rm GW} \approx \tilde{\sigma}_{\rm GW}(m_B)\left[\frac{\mathscr{I}\mathscr{N}}{\tilde{\mathscr{I}}\tilde{\mathscr{N}}}q\left(\frac{3q}{2+q}\right)^{1/7}\left(\frac{2+q}{3}\right)\left(\frac{1+q}{2q}\right)^{1/3}\right] \qquad q \equiv \frac{m_A}{m_B}$$



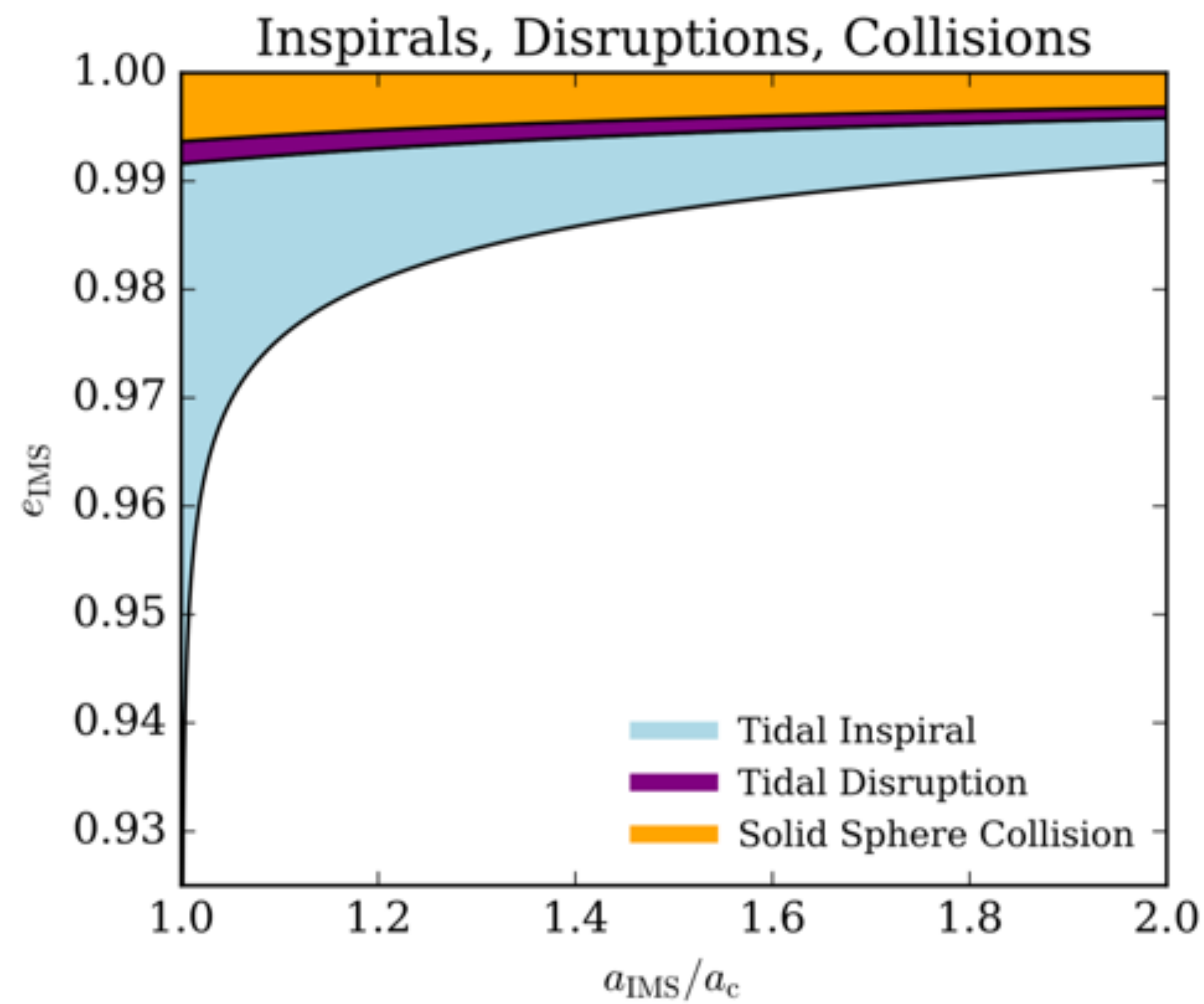


WD-NS tidal inspirals

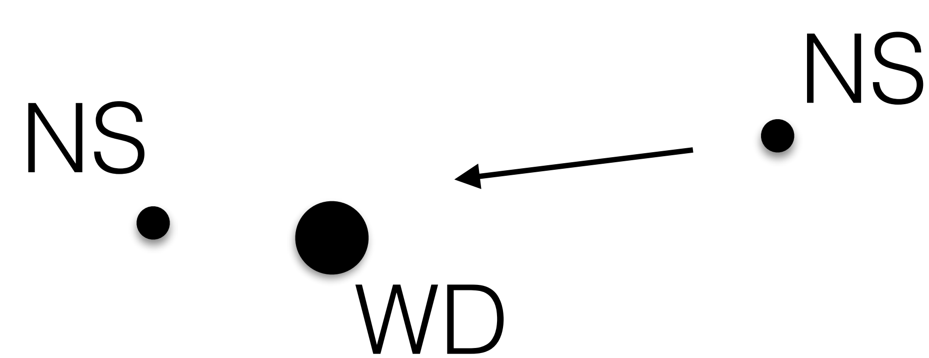




Orbital phase-space



The inspiral region increases relative to collisions as the SMA increases.

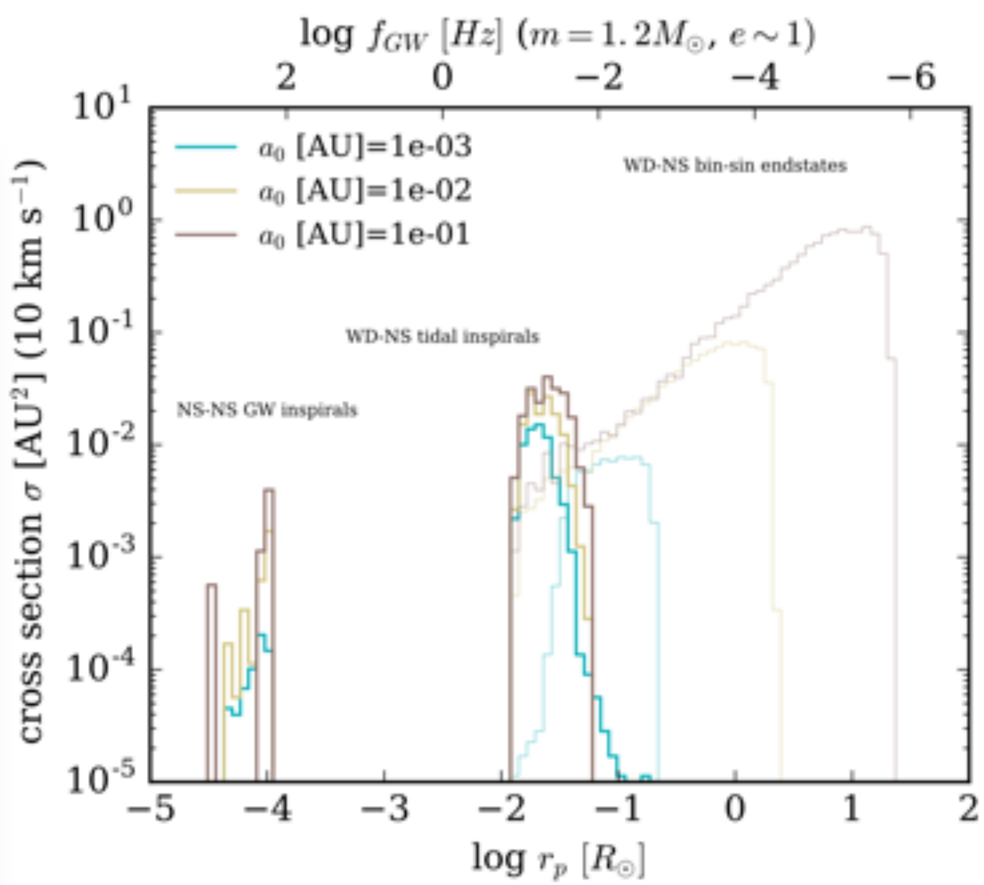


Conclusions:

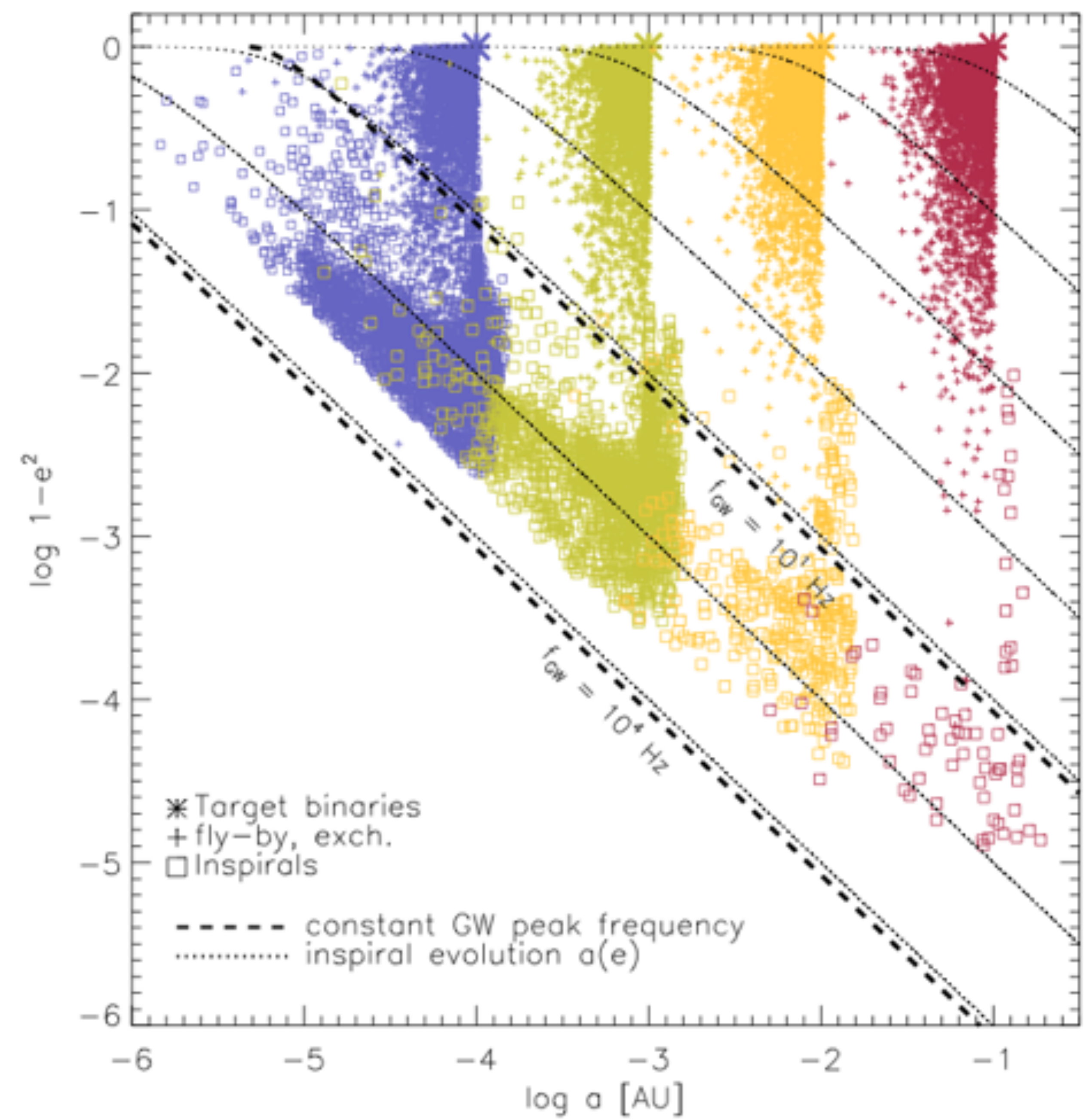
-The main effect from tides and GR is the formation of inspirals.

- Inspirals can dominate over collisions.

- The more compact an object is compared to the orbit, the more inspirals form relative to collisions.



High eccentric inspirals



Tides and GR are very important ingredients for the formation of high eccentric transients!